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A WATER QUALITY MODEL OF THE SACRAMENTO-SAN JOAQUIN DELTA

Water Resources Engineers, Incorporated
Lafayette, California

June 1965

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WRE

A WATER QUALITY MODEL
OF THE
SACRAMENTO-SAN JOAQUIN DELTA

REPORT OF AN INVESTIGATION
CONDUCTED FOR THE
DIVISION OF WATER SUPPLY AND POLLUTION CONTROL
REGION NINE
UNITED STATES PUBLIC HEALTH SERVICE

BY

WATER RESOURCES ENGINEERS, INC.
LAFAYETTE, CALIFORNIA

JUNE 1965

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LIST OF SYMBOLS

Symbol	Description	Dimensions*
A	Channel cross section	L^2
A_j^*	Surface area of the junction	L^2
A_t	Advective transport rate	$L^3\ T^{-1}$
α	Celerity of shallow water wave	$L\ T^{-1}$
b	Channel width	L
c	Concentration	dimensionless or $F\ L^{-3}$
c_s	Saturation concentration	dimensionless or $F\ L^{-3}$
C	Total pollutant load	F
C_a	Advected pollutant load	F
C_A	Surface area factor	dimensionless
C_b	Decayed pollutant load	F
C_d	Diffused pollutant load	F
C_e	Pollutant load, import or export	F
C_n	Any constant	
C_o	Oxygen Transfer	F
D_t	Diffusional transport rate	$L^3\ T^{-1}$
D_M	Molecular diffusivity	$L^2\ T^{-1}$
δ	Liquid film thickness	L
E	Rate of energy dissipation	$L^2\ T^{-3}$
g	Acceleration of gravity	$L\ T^{-2}$
γ	Specific weight	$F\ L^{-3}$
H	Head at channel junction	L

Symbol	Description	Dimensions*
i	Channel designation	
j	Junction designation	
K	Frictional resistance constant, Manning	
K_d	Eddy diffusion coefficient	$L^2 T^{-1}$
K_L	Liquid film coefficient	$L T^{-1}$
K_1	BOD reaction velocity constant	T^{-1}
K_2	Reaeration coefficient	T^{-1}
L	Biochemical oxygen demand, BOD	dimensionless
L_a	Ultimate first stage BOD	dimensionless
L_e	Statistical mean eddy size	L
M_t	Total transport rate	$L^3 T^{-1}$
n	Manning's roughness coefficient	
ν	Kinematic viscosity	$L^2 T^{-1}$
Q	Discharge	$L^3 T^{-1}$
Q_n	Algebraic flow rate, channels plus external sources	$L^3 T^{-1}$
r	Frequency of film replacement	T^{-1}
ρ	Fluid density	$F T^2 L^{-4}$
S	Slope of energy gradient	dimensionless
σ	Surface tension	$F L^{-1}$
t	Time	T
T	Temperature	
θ_K	Oxygenation rate characteristic	dimensionless
$\bar{V}$	Mean channel velocity	$L T^{-1}$
V	Velocity	$L T^{-1}$

Symbol	Description	Dimensions*
$\forall$	Volume	L^3
x	Distance along channel	L
X	Channel segment length	L
y	Channel depth	L

* L = Length, T = Time, F = Force

I INTRODUCTION

BACKGROUND FOR INVESTIGATION

The Sacramento-San Joaquin Delta, the confluence of the two major river systems of California's Central Valley, has been the subject of near continuous investigation since the advent of a general plan for the development of one of the State's greatest resources; water. Most of this study has been focused on a variety of schemes for the transference of water through the Delta Region to connect the principal elements of the impoundment and conveyance system designed to achieve a redistribution of water more in keeping with present and future need. Each of the schemes purported to solve the primary problem while minimizing detriment, or even enhancing, certain water uses within the Delta. Not withstanding the political problems involved, the aim of each proposal was to serve the worthy goal of maximizing benefit for all citizens of California.

While over the years many ideas were advanced, few of these gave any serious attention to matters of water quality except the ever-present problem of salinity control, around which much debate developed. Moreover, even in the face of potential salinity intrusion or modification of the natural equilibrium between the tributary river systems and the receiving embayments, no satisfactory methods were developed to adequately characterize the quality change which might be expected with each of the development alternatives. Field studies served to collect much valuable data but information gathered was generally limited in scope and evaluation was confined to empirical relationships or graphical depiction.

Some of these shortcomings can be better understood when one appreciates the complexity of the system of channels and islands which comprise the Delta and the technical difficulty of predicting its hydraulic behavior, even apart from its water quality behavior. A fairly simple description of the Delta as shown in Figure 1 acknowledges that it encompasses nearly 400,000 acres of irrigable farm land situated on more than 50 islands, is surrounded by more than 700 miles of channels, serves as a navigation link for the ports of Sacramento and Stockton, provide a

FIGURE - I

SACRAMENTO - SAN JOAQUIN RIVER DELTA

spawning ground for important fisheries, and serves as a recreational resource for hundreds of thousands of Californians. It is subject to tidal influence in its entirety, receives controlled and uncontrolled drainage from the Central Valley, an area which yields about 20 percent of all agricultural produce of the United States.

Currently, various attempts are being made to develop better analytical tools for study of the Delta. These include an electrical analog of the Delta and Upper San Francisco Bay and the extension of the physical model of Corps of Engineers to add the Delta system of channels. In their own ways, each of these tools has certain advantages over the other but both fall short in their capability for investigation of the water quality problems of the Delta.

Basically, there are at least two problems to be solved. First is the problem of representing the hydraulic behavior of the prototype including changes in tidal elevation, velocity and discharge. Without achieving this step the solution of the second problem (or group of problems) dealing with water quality is not possible although the hydraulic model or analog may, for certain special purposes, be an end in itself.

A study of water quality changes in an estuarial system requires representation of the complex processes of mixing and exchange which take place as water and pollutants are circulated under hydrologic and tidal influences. Neither the physical model nor the analog is presently capable of satisfying this essential requisite. Although the analog technique may potentially serve for certain special water quality studies it becomes prohibitively complex and costly for a system like the Delta and for the wide variety of complex problems which demand attention. The physical model, because of its gross distortion and its inability to satisfy simultaneously two or more laws of similitude, can never hope to model mixing phenomena.

The most attractive approach to resolution of this dilemna is the development of a mathematic model of the Delta System which would be comprised of two components "built" on the same basic structure. One of these components would deal with the hydraulics of the system and the other, using results provided by the first, would treat the water quality relationships. In this form the components actually would be separate models in their own right.

Each could function independently, but the information requirements for the water quality model should be such that for this type of investigation the most efficient operation would be obtained by conjunctive use. Both models would be "operated" with the aid of the digital computer for the solution of any specific problem for which the boundary conditions could be specified.

This report deals with the development of such a mathematical model of the Sacramento-San Joaquin Delta, particularly with the second component, that dealing with water quality. The first component - the hydraulic model - was developed earlier by Water Resources Engineers, Inc. of Lafayette, California, with the collaboration of the Division of Water Supply and Pollution Control, Region IX, U. S. Public Health Service. It served as the essential base for the water quality model and was used frequently throughout the investigation.

SCOPE AND OBJECTIVES

On June 29, 1964 Water Resources Engineers, Inc. entered a contract (No. PH 86-64-145) with the Bureau of State Services, Division of Water Supply and Pollution Control, Public Health Services to provide professional services for the development and evaluation of a mathematical water quality model of the Sacramento-San Joaquin Delta. In part the contract stipulates that . . . . "The completed model will reproduce the hydraulic behavior of the system and will provide quantitative information on the changes in water quality related to the hydraulic properties and to the major physical, chemical, and biological factors which can alter quality. The model will be capable of accounting for the effects on water quality of the following items:

1. Changes in quality of water delivered to the Delta at various points on its periphery.
2. Withdrawals of water at various levels of quality for use within the Delta itself, or for export.
3. Evaporation and consumptive use within the Delta.
4. Advective transport of water within the Delta channels as a result of inflows, extractions, and tidal induced currents.
5. Oscillatory tidal action with associated fluctuations in channel velocity.

6. Eddy diffusion related to velocity and channel configuration.
7. Addition of concentrated pollutants or contaminants at various points within the Delta.
8. Decay, destruction, or physical loss of pollutants or contaminants within the Delta.
9. Mass transfer through water surfaces (i.e. reaeration or deaeration).

The completed model may be either a single model which solves the hydraulic and quality problems simultaneously or a dependent quality model which uses, as certain of its fixed conditions, the results of a prior solution of a given hydraulic problem.

The completed model should be capable of solving such specific problems as the following:

1. Time history and distribution of a conservative or non-conservative pollutant through the network of Delta channels.
2. Sea water intrusion due to reduced outflows, excessive diversions, or channel blockage.
3. Effects of San Joaquin Valley Drainage discharges on quality of various outflow diversion locations.
4. Location of critical areas of pollution under various Delta development and water transfer plans."

As will be noted, WRE was given the option of developing either a combined hydraulic and water quality model or a dependent quality model. It chose the latter option, using the previously developed hydraulic model as a base.

ORGANIZATION

The development of the water quality model, described herein was accomplished by WRE personnel working in close collaboration with the Water Resources Section of DWS and PC, DHEW, Region IX, Public Health Service. Dr. G. T. Orlob served as principal investigator, Dr. Robert P. Shubinski was responsible for development of the computer simulation of the model, and Mr. M. R. Lindorf coordinated all engineering activities for the project. Mr.

James C. McCarty, Chief of the Water Resources Section served as Project Officer for the PHS. His engineering staff provided basic data input for the water quality problems to be studied with the model. Overall project direction was the responsibility of Mr. Paul W. Eastman, Regional Program Director, Division of Water Supply and Pollution Control, Region IX. Computer services were provided to the project by arrangement with C.E.I.R., Inc. of San Francisco, California.

ACKNOWLEDGEMENTS

Water Resources Engineers, Inc. wishes to acknowledge its appreciation for the splendid cooperation and support it received from personnel of the PHS Region IX Office, in particular, that of Mr. Paul W. Eastman and Mr. James C. McCarty, without whose encouragement and confidence the development of the model would not have been possible.

II HYDRAULICS OF THE DELTA NETWORK

While the development of the hydraulic model, per se, was not a part of the contract which is reported on herein, it does constitute an essential component for investigation of water quality problems of the Delta. Therefore, it is appropriate as background for discussion of the water quality model to review briefly the basis for selection of the mathematical model as such, the hydraulic behavior and characteristics of the Delta system, and the development and capabilities of the WRE tidal estuary network model itself.

SELECTION OF MODEL TYPE

It has been noted previously that several types of model representation of the Delta are possible or have been attempted. Each of these, the physical model, the electrical analog, and the mathematical model, has its limitations and advantages. It is considered desirable at this juncture to indicate what these are and to show justification for the approach adopted.

Physical Models

A physical model is most often justified on purely technical grounds when the hydrodynamic forces are applied in the prototype in such a complicated way that a direct analytical solution is wholly impractical. This may often be the case in harbors or estuaries which have widely variant depths and very irregular boundaries; or in those instances where a man-made structure is introduced into a natural environment. It must be recognized, however, that even for these cases not all of the laws of dynamic similitude can be satisfied simultaneously and while the model may correctly represent some natural phenomena it may be wholly incapable of representing others.

This limitation can best be appreciated when one examines the relative importance of some of the more significant forces which must be accommodated in the typical model. In modeling an estuary for example, it is obvious that gravitational and inertial forces are especially important. The customary practice uses

the ratio of these forces, designated as Froude's Number, as the criteria to be satisfied between model and prototype, that is, the ratio must be the same for both. Therefore, having adopted a suitable linear scale ratio for the model and prototype, it is possible to determine the ratios for other characteristics, such as area, volume, velocity, discharge, and time.

But other forces, which may also be important, may not be kept in the same proportion between model and prototype. In order to avoid a compromising influence of such forces as those of viscosity and surface tension, for example, the model may be deliberately distorted or modified in such a way as to make these forces unimportant. While it may be somewhat of an understatement of the entire technical justification, this is an important reason why the Corps of Engineers' model of San Francisco Bay has a vertical scale ratio of 1:100 and a horizontal scale ratio of 1:1000.

Once constructed to scale a physical model must be further adjusted, that is, "verified", to behave as the prototype. This process, which is often long, involved, and without rational basis, entails modification of bottom roughnesses in the model in such a way that tidal velocities and elevations are made to agree through the scale relationships with those of the prototype for selected tidal conditions and locations. To accomplish this modification strips of copper embedded in the model are tilted upward into the flow to angles and lengths which produce the desired effect on the characteristic being verified. By trial and error roughness may be increased or decreased until the appropriate combination is found. Models constructed in this manner, such as that of San Francisco Bay, have been successfully employed in the study of general circulation induced by tidal action.

It is readily apparent from the nature of the physical model, its distortion and widely varying roughness, that it cannot be expected to represent at the same time processes which are gravitationally oriented and those which are related more to the frictional properties of the system, since these latter have been arbitrarily distorted in the model. Accordingly, the model is, a priori, incapable of representing turbulence and dispersion phenomena.

It is true that estuarial models have been forced to represent certain gradients of salinity between the landward and seaward boundaries. These have

generally been accomplished through the same sort of "verification" process described previously but without any real rationale and at the sacrifice of faithful representation of other qualities of water movement. While some qualitative information may be developed from the model, the fact that the processes of mixing in the model and prototype are not actually linked through a law of similitude tends to diminish somewhat the value of such observations. The mere fact that diffusional processes are influenced strongly by boundaries is a serious handicap to use of the model, which must of necessity be limited in size.

As devices for the study of water quality and pollution problems physical models may serve a useful purpose in their ability to show visually to the observer what the general circulation pattern might be. They leave much to be desired on technological grounds for water pollution investigation and can seldom be justified, because of their high cost, for this purpose alone.

Analogs

An analog is a device which simulates the behavior of the prototype through the use of a system of components, each of which performs a function "analogous" to some element of the prototype. The most common analogs for the flow of water make use of the properties of electrical circuits in which voltage drop, current flow, and conductor resistance serve as counter parts to energy loss, discharge and conduit resistance. The fact that the basic laws governing flow in simple conductors and conduits are not functionally identical has been overcome by the development of so-called "square law" resister elements. Other unique properties of fluid systems such as storage, wave propagation, and wave attenuation have been conveniently represented by electrical systems.

The most notable examples of electrical analogs are those for solution of the Hardy-Cross problem of flow distribution in pipe networks. However, a much more sophisticated analog for open channel networks bears directly on the problem at hand was developed for Upper San Francisco Bay and the Lower Delta by Professors J. A. Harder and H. A. Einstein of the University of California. The Harder Analog is said to be capable of reproducing tidal elevation and flow histories for an estuarial system comprised of a large number of elements, perhaps as many as 100 depending

on the detail required. It does not purport to be an analog of water quality phenomena nor to represent the processes of mixing and dispersion.

Analogs have at least one important advantage over other types of prototype representation. They respond virtually instantaneously (at the speed of electron flow) to changes in input conditions. This enables the operator to scan rapidly through a large number of problems (although the number of possibilities may be enormous) and to elect those most promising for additional concentrated study. With the exercise of a certain amount of electrical genius electrical analogs may be made to accept continuous records of tidal input and to reproduce continuous output for selected stations. This capability may prove useful in study of long-term operation of systems or in developing data for stochastic analysis.

Electrical analogs for large systems may be costly to build, although cheaper than physical models. They are much more exacting in operation and because of their intricacy they may offer special problems in maintenance. Moreover, the time required for their development and construction is probably not much less than for a physical model of the same system. The Harder Analog was developed over a period of about 5 years, during which time many new techniques of analog construction were developed. Actual construction time for the most recent version was about 1.5 years.

An important distinction between an estuarial analog and a physical model is that the analog is developed from a general mathematical formulation of the problem. The physical model does not require this formulation; in fact, is most suited to cases where such formulation is impossible or impractical. A danger exists with the analog; failure to correctly formulate the problem leads to an incorrect solution, one which does not always indicate the fallacy of original assumptions. This is a shortcoming of all mathematically based approaches but seems to be more critical in the analog because its response does not permit discriminate diagnosis of the source or location of error.

In summary, analogs of flow systems have advantages over physical models in their lower cost and rapidity of response. Their disadvantages lie in their greater technical complexity and in the limitations imposed by the use of

sophisticated electrical components to represent the complex behavior of the prototype. While theoretically it is possible to construct an analog to represent almost any physical process, considerable ingenuity, time and expense may be necessary to achieve the desired result. Frequently this greater expense and delay in achieving a workable analog may not be compatible with the importance of the results achieved.

Mathematical Models for Digital Computation

A mathematical model is a functional representation of the physical behavior of a system or process presented in a form available for solution by any acceptable method. In its simplest form the mathematical statement of a process consists of an input, a transfer function, and an output. The output from a given system or component of a system is taken to be related to the input or some function of the input; i.e., the transfer function.

A mathematical model of a system, such as the Sacramento-San Joaquin Delta, consists of a series of elements connected in such a way that the output from one element becomes the input to the next and so on. Each input is operated on by the transfer function for the element and through a succession of steps in time and space, the entire process can be followed. While it is possible for each element of the system to be described by a single function or set of functions and to obtain direct continuous mathematical solutions for its behavior such information may not be sufficient to completely characterize a system of elements. One advantage of the mathematical representation is that it permits discretizing and more detailed characterization of the prototype. Characterizing the behavior of a complex entity such as the Delta by simply defining an input and a single transfer function is an unquestionable absurdity. However, reducing this entity to a system of elements for which the separate transfer functions may be more satisfactorily applicable, permits the interaction of these elements and can result in a closer representation of a true prototype behavior.

While it is often possible to write the differential equations which characterize a particular phenomena under study, it is not always possible to obtain a

direct solution. Occasionally, direct solutions for the simplest of systems are available, but when the boundary conditions required by real circumstances become excessive or complex in themselves, there may be no direct mathematical means of obtaining a solution. Under these conditions it is convenient to resort to numerical methods which require discretizing the system so that the boundary conditions for each element may be defined. By proceeding through the system from element to element using the output of one as the input to the other and satisfying common boundary conditions in succession, the complex behavior of the system can be evaluated.

If, as in the case of the Sacramento-San Joaquin Delta, the phenomena to be studied are time dependent, it is convenient also to increment the solution temporally. Thus the system may be treated as being in static or dynamic equilibrium for some finite time interval instead of being continuously variant. Having removed another variable from consideration, a solution is now possible. Of course, the precision obtained in the result is dependent to some extent on the length of time interval selected and the rate of change of the phenomenon being studied. The greater the number of finite time intervals used over the total period of investigation, the more accurate the result can be expected to be.

While numerical methods are admirably adapted to discretized systems where the transfer functions may be taken as time independent over short time intervals, there are certain practical limitations in their application. For systems as complex as that of the Delta with its more than 130 channels subject to diurnal fluctuations of tide, it is easy to appreciate that the number of individual computations necessary would impose an impossible burden if the investigator were obliged to carry out these calculations in a conventional manner. The advent of the electronic digital computer makes possible the attainment of a reasonable solution within a practical range of cost. It can readily be seen that there is some desirable optimum achieved between the demand for accuracy and the burden of additional cost which may be proportional to both the number of elements and the number of separate time increments required for a complete analysis of the system. Occasionally certain other considerations, such as stability of the solution, may have an important bearing on the optimal size of element and interval of time.

Experience has shown that other advantages of the mathematical model approach with solution on the digital computer are substantially reduced cost and development time over those required for either the analog or the physical model. In the case of the Delta it is estimated that the cost of mathematical computerized model development may be as little as one-fifth that of the analog. The time required for perfection of the mathematical model solution with the computer could be as little as one-third that required by the analog. Against these advantages one must consider the somewhat higher cost of operation for solution of a single problem on the computer. However, a carefully planned program of investigation can usually reduce these costs by selecting only those problems worthy of investigation with the model.

HYDRAULIC BEHAVIOR OF THE DELTA

Physical Description

The Delta is comprised of a complex of channels of irregular form which were developed initially by the natural meanderings of the two principal river systems through the detritus of erosion from the Central Valley. Subsequently these channels were improved as man became aware of the fertility of the Delta lowlands and proceeded to reclaim them and put them into agricultural production. Levees were constructed around certain tracts, usually natural islands between meander channels, and the protected land was drained and cultivated. The need to serve the interior of California with the products of international commerce led to further development of these channels as navigable waterways. Channels progressively became fixed in location and were maintained at more or less static cross-section geometry controlled by periodic dredging and levee improvement.

Channels in the Delta may be generally classified as broad and shallow. Widths of principal reaches range from 50 to several thousand feet while depths at mean tidal elevations range from 5 to 40 feet. Width to depth ratios are generally greater than 10 but may be greater than 100 in some of the larger lower reaches. In general, channels of the system may be regarded as two-dimensional, that is, the depth may be taken as equal to the hydraulic radius. Exceptions are a few of the

smaller channels with low width-depth ratios and considerable vegetation along the banks. However, hydraulically speaking, these channels are of minor importance. The approximation of two-dimensionality would not be expected to introduce significant error in the behavior of the entire system.

Due to the pattern established by natural meanderings of the rivers, reclamation of agricultural land and the infrequent inroads of flooding on the levee system, channels are connected in a somewhat random fashion. These junctions have tended, over the years, to assume hydraulically efficient configurations; that is to say, the processes of erosion and deposition have formed smooth bends and have filled in stagnant areas which contribute little to the circulation of water in the system. It might be expected that energy losses at junctions, although significant, would be minor relative to frictional losses in long channel reaches.

Channel Resistance

For purposes of practical study the system of interconnected channels may be represented as a reduced network of one-dimensional conduits such as that shown in Figure 2. The lengths of channels and junction points are selected so as to represent with reasonable faithfulness the real configuration of the system while at the same time eliminating excessive detail not consistent with the kind of information required of the model. For example, small scale networks or small flooded areas within the Delta could be replaced by single equivalent channels in such a way that large scale water movements in the system are not appreciably affected. This sort of arbitrary modification, simplifying and removing complexities which are not essential to resolution of the problem, requires that attention be given to the hydraulic properties of the separate channels.

Each of the channels shown in Figure 2 must be characterized as to its roughness and resistance to flow. For the purposes of the studies reported herein

FIGURE - 2

REDUCED DELTA NETWORK - STEADY STATE PROBLEM

Manning's equation, which may be written as

$$\frac{dH}{dx} = \left(\frac{2.207}{n^2 y^{4/3}}\right) \bar{V}^2 \qquad (1)$$

was adopted as the relation defining resistance to flow in terms of physical properties of the channel. In this equation the various terms are defined as follows:

$\frac{dH}{dx}$	=	energy gradient = slope of water surface
y	=	mean channel depth = hydraulic radius
n	=	Manning's roughness coefficient
$\bar{V}$	=	mean velocity in reach

In order to apply equation 1 it is usually necessary to assume the following:

1. that the discharge is steady; that is, it does not vary over some finite time interval, and
2. that the flow is uniform; that is, that the discharge and velocity do not vary along the axis of the channel during said time interval.

Implicit in these assumptions are the conditions that the cross-section of the channel is invariant during the interval and along the channel and that the slope of the energy gradient is the same as that of the water surface and the channel bottom. It is obvious that only under the most unique conditions can these conditions be satisfied and that treatment of cases such as that represented by tidal oscillations in the Delta Network will necessitate consideration of unsteadiness. In these cases however, the Manning relation can still serve to characterize the resistance properties of the channels.

Selection of an appropriate resistance coefficient is a matter in which experience and judgement play important roles. In the absence of direct measurements, which are virtually impossible for the case in question, one must adopt values which are consistent with observations made for similar channels under comparable circumstances. For the Sacramento-San Joaquin Delta System values of n ranging from 0.025 to 0.043 were assigned to the various channels depending on the size of channel and the degree of channel maintenance. As in the other instances where estimations or assumptions had to be applied to achieve a practical result, the real proof lies in simulating correctly the prototype behavior.

Description of the Delta Hydraulic Model

The Delta Hydraulic Model was constructed in a succession of steps of progressively increasing difficulty. Appreciating the inherent problems in attacking a system of such complexity as the Delta and representing it mathematically in all of its intricacy, it was decided to begin initially with the simplest possible representation and to proceed step by step toward the ultimate goal, a model fully capable of representing the dynamic character of the system.

Examination of the reduced Delta network as shown in Figure 2 suggested the possibility of treating this network of inter-connected open channels in a manner similar to that employed for the solution of pipe networks by the Hardy-Cross method. It was recognized at the outset that this approximation was a crude one and could not be expected to yield fundamentally correct solutions, but it was believed that much could be learned from this approach. Moreover, it was anticipated that the development cost for this special case of the Hardy-Cross problem, for which a unique method of solution had already been developed, would be fairly low. Subsequently, as experience was gained in operation of the Delta network under static conditions, the dynamic solution could be developed and applied.

Steady State Solution

The reduced Delta network shown in Figure 2 is comprised of 134 separate channels interconnected at 93 joints. Each of these channels may be characterized hydraulically, as in Equation 1, by its length, cross-section, and roughness, and is capable of transmitting flow in both directions depending on gradients induced by

assumed steady inflows. In the initial study of a steady state condition, properties of the channels were defined for mean half-tidal elevations based on direct field observations performed by the Department of Water Resources personnel or on data taken from published navigational charts. While for the first approximation values of Manning's "n" were selected as constant for all channel sections at 025, it soon became apparent that there was need to include consideration of variation in channel roughness from point to point within the Delta. Taking into account variations in size of channel and qualitative variations in channel roughnesses, values of n were selected as ranging from 0.025 to 0.043. In a steady state solution channel cross-sections were taken as constant and independent of flow.

The WRE Distribution Network Program* designed for the Delta was supplied with basic data on channel properties, inflows and withdrawals, and assumed potentials at all junctions. Machine computation resulted in a balanced hydraulic system, reporting corrected potentials at all joints and flows in all channels. Iteration was continued through sufficient cycles to guarantee balance of flows at each joint to an error within one percent of the absolute sum of all inflows and outflows.

Because of the complexities of the system and the preliminary nature of the studies of the steady state solution, no attempt is made herein to analyze in detail the changes which occurred with modifications in inflow or network configuration. For reference purposes, however, Figure 3 is included. This example illustrates qualitatively the computed direction of flow in the separate channels for two different flow conditions. Table 1 reports computed flows in selected channels designated on the figure.

* Water Resources Engineers Distribution Network Program

In August 1960 the flow to the Delta from the San Joaquin River was estimated at approximately 100 cfs. For this condition the model indicated upstream flow in the River in the vicinity of Stockton. In view of the conditions which appeared to have existed near Stockton, it was considered appropriate to examine briefly the effects of increasing the San Joaquin River inflow. Additional computer runs were made with conditions identical to those of August 1960 except that the San Joaquin flow was increased to 1200 cfs instead of the 100 cfs which actually prevailed. Figure 3 indicates those channels in which a flow reversal occurred and Table 1 summarizes flows for selected Delta channels under these two different inflow conditions.

Dynamic Steady State Solution

As is the case for many physical problems, the solutions to the equations which govern the propagation of tidal waves in open channels have been obtained in closed mathematical form only for highly idealized conditions which seldom exist in nature. Even those solutions which do exist have been obtained by imposing certain simplifications and restrictions which make it difficult to relate them to real situations. Hence, it is appropriate to adapt the equations for numerical solution and discretize the system to facilitate this approach to a practical solution.

Flow in the Delta can be treated as a one-dimensional wave propagation problem. The length of the wave generated by tidal action is much greater than the depth of the channel, so that the applicable equations are those for long waves and shallow water. A characteristic of such waves is that the motion of water particles is not confined to the region near the surface, but extends to the bottom. Hence bottom friction effects the behavior appreciably. Moreover, friction and velocity are not linearly related, so the governing equations are non-linear. Closed form solutions exist, but only for very simplified cases which are unlike the complex network of the Delta.

In formulation of the problem of wave propagation in a simple channel, it is necessary to satisfy both the equations of motion and continuity. The equation of

FIGURE - 3

DIRECTION OF FLOWS - STEADY STATE PROBLEM

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motion is given as

$$\frac{\partial \bar{V}}{\partial t} = -\bar{V}\frac{\partial \bar{V}}{\partial x} - K\bar{V}\bar{V} - g\frac{\partial H}{\partial x} \qquad (2)$$

where $\bar{V}$ = velocity in the stream, positive in the direction of increasing x,

H = height of water surface above some horizontal datum,

g = acceleration of gravity,

K = frictional resistance factor derived from Manning's equation (1),

t = time

and x = distance along the axis of the stream.

The equation of continuity is given as

$$\frac{\partial H}{\partial t} = -\frac{1}{b}\frac{\partial}{\partial x}(\bar{V}A) \qquad (3)$$

where A = cross-sectional area of the channel and b = mean channel width.

Equations (2) and (3) are based on the following assumptions:

1. Acceleration normal to the x axis is negligible.
2. The bottom of the channel is level.
3. The wave length is at least twice the channel depth.
4. Coriolis acceleration and wind forces are negligible.
5. The channel is straight and its cross section does not vary with length.

In reality, the velocity from point to point throughout the cross section of the channel is not constant. This velocity variation accounts for dispersion of a pollutant transported through the channel. However, recognition of this fact in the above equations violates their one-dimensional nature. This dilemma is avoided by considering $\bar{V}$ to be the mean velocity at a section.

Each of the terms in the governing equations can be physically identified. In equation 2 the left hand side is the velocity gradient with respect to time. The right hand terms represent respectively: the Bernoulli acceleration or rate of momentum change by mass transfer, the frictional resistance, and a gravity driving force or potential difference. The indication of absolute value for the velocity guarantees that

the friction force opposes the water motion at all times. The left hand side of Equation 3 is the potential gradient with respect to time while the right hand side represents the change in storage per unit width of channel in the system with respect to distance along the channel.

Although the velocity in each of the channels is a vector quantity, no effort is made to consider the change of direction in velocity which occurs at bends and curves in the system. Such vector effects can be neglected without serious error because of the magnitudes of the velocities encountered are small in comparison to the physical dimensions of the channel. Actual curvatures are never as sharp and as sudden as those in the idealized system. As concerns the matter of added frictional resistance due to bends or junctions in the system, it is appropriate to account for these by using an equivalent length of channel which would produce a frictional loss in accordance with the term of equation 2 which is equal to that achieved in the real system between junction points.

It has been established that solution of Equations 2 and 3 for the Delta system must be by numerical techniques. This requires that the equations be rewritten in finite difference form as follows: The equation of motion becomes

$$\frac{\Delta V_i}{\Delta t} = - V_i \frac{\Delta V_j}{\Delta x_j} - K|V_i|V_i - g \frac{\Delta H}{X} \qquad (4)$$

where the subscript i denotes the channel under examination. The equivalent channel length is designated as X and ΔH is the potential difference between the ends of the channel. The quantity Δx_j is the length from the mid-point of the channel directly ahead of channel i to the midpoint of the one directly behind it. In the case of branching channels an average length is used. Similarly, ΔV_j is the corresponding difference in velocity. The equation of continuity becomes:

$$\frac{\Delta H}{\Delta t} = \frac{Q_{net}}{A_j^*} \qquad (5)$$

where the subscript j refers to the junction under consideration. The term Q_{net} is the algebraic flow rate, both from channels and external sources. A_j^* is the surface area of the junction.

It is appropriate in the solution of these equations that certain properties be assigned to channels and other properties to junctions. For obvious reasons those properties associated with the transmission of flow from point to point in the system are associated with channels while changes in elevation and volume are identified with channel junctions. Each junction in the Delta network was assigned a volume equal to one-half the volume of each of the inter-connected channels.

The solutions of Equations 4 and 5 can be accomplished by a process of numerical integration carried forward in time. For a small system, composed of a few channels or channel segments, this might be done manually, but a system with the physical size and complexity of the Delta network can only be solved with the assistance of a digital computer. A general purpose program which will be described subsequently in this report, was written for the IBM 7094. It was structured in such a way as to accomodate a system of channels composed of as many as 1,000 separate elements connected at 750 junctions. Provision was made in the program to accomodate prescribed tidal inputs at five boundary locations.

The integration method used was a modified Runge-Kutta procedure, in which only two subintervals were used instead of the customary four. In operation of the program Equations 4 and 5 are solved alternately beginning with an assumed condition at time t. A solution is first obtained for velocity, flow, and head for the half-step interval, $t + \frac{\Delta t}{2}$. Taking the half-step values as the average throughout Δt, it is then possible to obtain velocity, flow, and head for the full step, $t + \Delta t$. As a part of a computation for each step changes in cross-sectional area and width, i.e. changes in storage, are determined for each channel or channel segment.

Computation proceeds sequentially through a desired period of time at intervals Δt until a suitable record of the behavior of the various elements of the system has been obtained. Experience with the program demonstrated that convergence on the correct system behavior was rapid and usually occurred in less than 10 hours of prototype time from the initiation of computation. Hence, for the majority of cases studied, simulation of prototype behavior for about 24 hours was sufficient to include the full cycle (12.5 hours) under dynamic equilibrium conditions.

Results of the integration delivered as print out from the computer are complete stepwise records of velocity, flow, and elevations in the entire system. This record is stored on magnetic tape for subsequent use in the analysis of water quality.

A selection of time interval to be used is not completely arbitrary and is an important determinant in the configuration of the reduced network subjected to analysis. As will be seen from examination of Figure 4 the configuration of the idealized system for the dynamic steady state solution is substantially more complex than for the steady state case. The added complexity for the solution of the transient flow problem develops from difficulties in achieving a stable solution unless the space interval Δx and the time interval Δt are related to the celerity of the waves, α, such that

$$\Delta x = \alpha \Delta t \qquad (6)$$

The mathematics of the Delta problem are sufficiently similar to the problem of classical wave propagation to make the question of instability worthy of examination. It was found by experience that the instability did exist, but that a precise relationship such as that suggested in Equation 6 governing space-time interaction could not be defined for the Delta network. As a practical expedient, however, Equation 6 was used as a guide to selection of appropriate channel length and integration time. For practical reasons, both Δx and Δt should be made as large as possible, consistent with the degree of precision required in the solution. Obviously, Δx could not be larger than the length of any one channel in the reduced Delta network shown in Figure 2. For the shortest channel of the reduced network it was determined that Δt should be approximately 150 seconds. This time interval was adopted for the program and was found to give satisfactory accuracy without the risk of instability.

Having adopted Δt and noting that the celerity of the wave was approximately proportional to the square root of the depth of channel, it was possible to further modify the Delta network by segmenting the separate channels at a number

FIGURE - 4

REDUCED DELTA NETWORK
DYNAMIC STEADY STATE PROBLEM

of arbitrary junctions. As it was finally reduced for the transient flow solution, the Delta consisted of 625 channel segments connected at 572 separate junctions with a single tidal input at Pittsburg. The modified network for the dynamic steady state solution is illustrated in Figure 4.

Verification of a Mathematical Model of the Delta

As in the case of physical models or analogs, the mathematical model can not be considered a satisfactory representation of the hydraulic behavior of the Delta until it is verified against the prototype. In the case of the Delta, the complexity of the system and the paucity of direct observational data on its hydraulic behavior make a satisfactory verification somewhat difficult to achieve. Never-the-less, while information concerning tidal velocities and discharges is almost non-existent, there is considerable information concerning the influence of tides on the Delta. An extensive network of tide gage stations has been installed and maintained and data from these stations was available for verification of performance of the model. Results of a few typical comparisons between tidal records on the prototype and the model are presented in Figure 5. Reference is made to Figure 4 for the location of these comparison stations.

A few brief comments concerning the behavior of the model in reproducing tidal fluctuations at these stations is appropriate. It will be noted in examination of Figure 5 (a), the tidal record for Antioch, that the computer output duplicates almost exactly the recorded tide at that station. This is more or less to be expected because of the proximity of Antioch to the lower boundry of the model where a tidal input of similar form was admitted. In general, verification in the lower Delta was excellent and no difficulties were experienced in producing valid tidal fluctuations and in determining associated velocities and discharges.

It will be appreciated that in order to initiate study with the model, assumptions must be made as to the water surface elevations at all junction points. Obviously, many of these assumptions are not in close agreement with actuality and some time is required for the model to generate a tidal wave which, in fact, agrees with the prototype. Comparison of prototype records for Antioch with those for Mossdale Bridge, which is situated at the south extremity of the Delta on the San Joaquin River, indicates

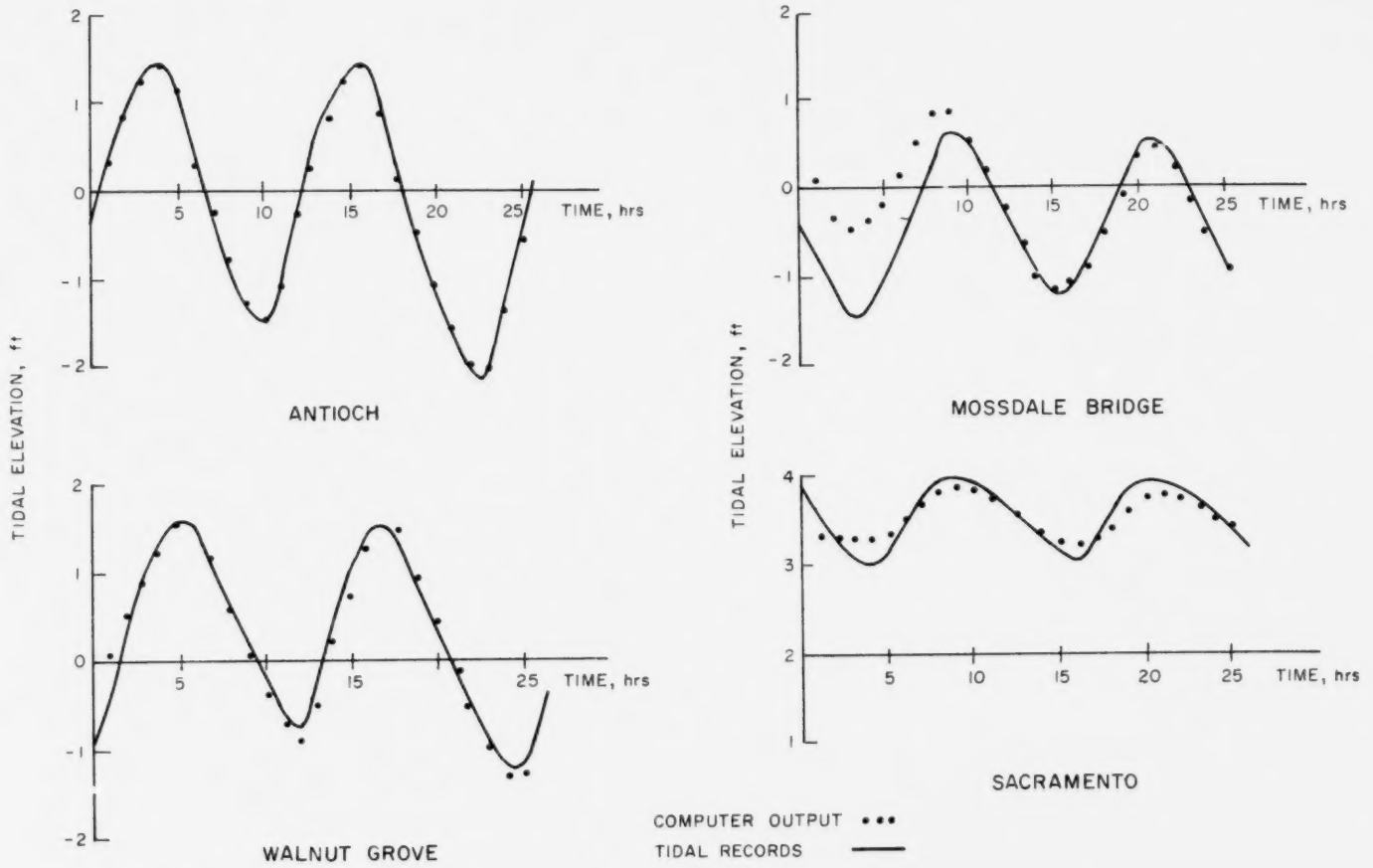

FIGURE-5
MODEL VERIFICATION OF TIDAL HISTORIES

indicates a phase lag of approximately 5 hours between these two stations, that is, about 5 hours is required for a disturbance generated at Antioch to pass through the Delta and arrive at Mossdale Bridge. Since the model is designed to respond to disturbances produced at the lower end of the Delta in a manner similar to the prototype and since wave propagation through the model proceeds in accordance with the theory of shallow water waves, it is to be expected that the Mossdale Bridge gage would not feel the real effect of the tide at Antioch until some five hours later. In actuality, about 10 hours was required to establish a state of agreement between the prototype and the model. Subsequently the computed and observed curves were in generally good agreement as to phase and amplitude.

The Walnut Grove station, which is situated about 30 miles up the Sacramento River from Antioch showed a more rapid convergence on correct behavior, perhaps due to the fact that the initial assumed conditions were closer to the prototype elevations at the outset. The phase lag between Antioch and Walnut Grove was only about two hours and satisfactory agreement between model and prototype was obtained within three hours.

The station at Sacramento, which is located about 15 miles beyond the northern boundary of the Delta, shows a computed tidal elevation which has the correct phase relationship with respect to the observed curve, but which has a somewhat smaller amplitude. This discrepancy in amplitude, which may be attributed to boundary reflections and the fact that the Sacramento River was influenced by tide some considerable distance beyond Sacramento, was not considered serious in view of the fact that excellent verification was obtained for stations within the interior of the Delta.

ILLUSTRATIVE EXAMPLE

In order to illustrate the behavior of the mathematical model a selected example has been included in this report. The pertinent hydrologic, tidal, and water use information which served as boundary conditions are described as to general nature and sources as follows:

1. Inflows from the Sacramento, San Joaquin, Consumnes, Mokelumne, Dry Creek, and Calaveras rivers were those of August 1964.
2. Consumptive uses and agricultural returns within the Delta were based on estimates of the Department of Water Resources for typical August conditions.
3. Withdrawals at Tracy to the Delta-Mendota Canal correspond to August 1964 conditions.
4. Other withdrawals from the Delta correspond to August 1964 conditions.
5. Tidal conditions correspond to the mean diurnal fluctuation for a typical lunar cycle of the summer period.
6. Imports to the Delta include discharge from the proposed San Joaquin Master Drain (900 cfs) and the domestic sewage discharge from the City of Stockton (40 cfs).

As noted previously, the program generated histories of velocity, discharge, and head for each element of the Delta System. All essential computed data for a full tidal cycle were stored on magnetic tape for subsequent use. When required for purposes of evaluation of hydraulic performance of the system any of these data desired can be printed out as a permanent record.

Table 2 and Figure 6 summarize the general behavior of the Delta for this illustrative run and give specific output information for a few selected locations. No attempt has been made here to analyze or interpret this information, but only to illustrate the nature of the data that can be made available to the compatible water quality programs whose development is described in the following chapters.

TABLE I

STEADY STATE SOLUTION

AUGUST 1960 CONDITIONS

STATION	SAN JOAQUIN RIVER INFLOW 100 cfs	SAN JOAQUIN RIVER INFLOW 1200 cfs
1	6875	6875
2	4862	4864
3	1865	2422
4	3859	3990
5	945	817
6	2586	1618
7	189	165
8	1012	920
9	3158	2319
10	580	573
11	2148	1975
12	1828	1662
13	587	- 75 *
14	476	- 184 *
15	353	- 342 *
16	53	-1152 *
17	277	782
18	377	75
19	191	286
20	3240	3138

* Minus sign denotes flow reversal. See Figure 3.

TABLE II

DYNAMIC STEADY STATE SOLUTION

AUGUST 1964 CONDITIONS

STATION	MAXIMUM VELOCITY (fps) EBB	FLOOD	MAXIMUM DISCHARGE (cfs) EBB	FLOOD	NET DISCHARGE (cfs)
1	2.5	-2.2	208,000	-200,000	7100
2	2.0	-2.0	136,000	-146,000	300
3	0.7	-0.5	48,000	- 36,000	5000
4	1.4	0.6 *	10,300	4,600 *	8000
5	0.5	-0.2	2,510	- 840	920
6	0.9	-1.0	23,000	- 27,000	750
7	0.8	-1.3	7,200	- 12,070	2590
8	0.8	-1.6	5,200	- 11,400	3500
9	0.1	-1.0	160	- 1,800	810
10	0.9	-1.1	1,300	- 2,000	240
11	0.4	-0.7	800	- 1,450	284
12	0.5	-0.6	1,300	- 1,600	220
13	0.4	0.05 *	407	22 *	230
14	0.5	-0.4	1,370	- 1,090	200

1) Net discharge in direction shown in Figure 6, See Equation 26,

2) No flow reversal occurs.

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FIGURE - 6

DIRECTION OF FLOWS
DYNAMIC STEADY STATE PROBLEM
(AUGUST - 1964)

III WATER QUALITY CONSIDERATIONS

NATURE OF POLLUTION

Pollutional effects on the Delta are many and varied. However, in general these effects may be reduced to two distinct categories; those dealing with so called "conservative" pollutants, such as may result from agricultural drainage or the intrusion of seawater into the lower Delta and "non-conservative" pollutants which are related to the burden of organic material carried into the Delta through tributary streams or discharged from domestic or industrial scurces.

Conservative Pollutants

A conservative pollutant is one which, for purposes of the water quality model described herein, circulates within the defined domain remaining unchanged in quantity except by deliberate import or export. For example, total dissolved solids which are only moved from place to place within the system and which are not destroyed by any of the processes represented in the model, may be defined as non-conservative in the present context. They are not subject to chemical or biochemical degradation.

Non-Conservative Pollutants

A non-conservative pollutant is one whose concentration is affected, not only by the physical processes involved in movement of water in which the pollutant is carried, but which may also undergo decay or degradation as functions of time. In the present situation the most obvious non-conservative pollutant is domestic sewage discharged from the cities situated in, or tributary to, the Delta. While in a rigorous sense the term "non-conservative" is a misnomer, the implication in the present context is that the loss or sacrifice of material removes it from consideration as a pollutant per se. Hence it is not conserved insofar as the model is concerned, but is withdrawn once and for all from having any impact on water quality with the Delta. The mere process of extraction, of course, may have an impact on the quality of the water in the Delta but this effect can only occur at one point in time and space.

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Many water quality characteristics or parameters are non-conservative in that their concentration within the system depends upon their equilibrium with sources outside of the system. A good example is the oxygen resource. Through the processes of reaeration, oxygen from the atmosphere, which is not a part of the present model, may be brought into the system. Once within the system it may be acted upon by forces which tend to consume it. Biochemical stabilization of organic material, for example, requires that oxygen be extracted from the water for use by bacteria in biochemical oxidation processes. While technically, oxygen is still conserved it is no longer identified in its original state and in its present context would be considered "non-conservative".

DISTRIBUTION IN SPACE AND TIME

From the standpoint of evaluating the impact of various Delta water transfer schemes on water quality, it is important to be able to evaluate separately certain water quality parameters and the processes which affect them. Because of the interaction between water quality and other water uses which are space and time oriented, it is essential that the quality of Delta waters be completely defined both spacially and temporally.

The water quality model should have the ability to account for variations in the quality of Delta waters for a wide range of hydrologic and tidal conditions. As a practical matter, however, certain of these conditions are known to be critical insofar as quality is concerned and have been agreed upon by interested agencies as common bases for evaluation. Thus, combinations of extreme tidal conditions and unusual hydrologic circumstances can be selected as those most likely to produce critical circumstances insofar as any particular water quality parameter is concerned. For this reason, stream flow inputs to the model at its boundary, import and export within the Delta, and extractions of water by evaporation within the Delta confines are most conveniently treated as constant rather than variable.

It is difficult to ascertain in advance which tidal conditions are likely to produce the most critical water quality conditions within the Delta but since the range of tide in this geographical location is small the changes brought about by differences in diurnal pattern of tidal fluctuations are relatively unimportant. Tidal

fluctuation per se is, of course, of great significance in determining the residence time of a pollutant in a particular channel, especially when a combination of hydrologic circumstances results in making a particular reach either neutral or negative insofar as net excursion of water seaward is concerned.

It is obvious that tidal action alone can bring about the movement or circulation of pollutants throughout the tidal estuary. This action may accelerate greatly the mixing processes associated with fluid turbulence and the bifurcation and rejoining of bodies of water as they oscillate through the system of channels of which the Delta is comprised. A tidal impulse of completely regular character imposed upon the lower end of the Delta, an asymmetrical network, can be expected, in the absence of other influences, to induce an asymmetrical circulation of water within the Delta. This phenomena known as "tidal pumping" is an important determinant in establishing a temporal and spacial distribution of pollutants admitted to the system of channels.

TRANSPORT PROCESSES

Any material introduced as a pollutant into the open channels which comprise the Delta is moved from place to place by essentially two processes, advection and diffusion. In addition it is possible to extract water at any point in the system, carrying away pollution at the concentration existing at that point or to introduce at any particular point in the Delta, a quantity of pollutant. The ultimate distribution of a particular material in time and space within this complex system is determined by all of these processes acting simultaneously under the influence of oscillatorial and tidal action, hydrologic circumstances, and man controlled diversion and circulation. It is appropriate at this point to discuss briefly the theoretical nature of these transport processes and the practical means through which they may be represented in the model.

Advection

The term "advection" as used in the present context is simply the translation of a particular volume, mass, or characteristic of the water at a velocity defined for the particular point in space and time. Thus, if the characteristic identified in the water is a particular contaminant at a concentration, c, a quantity

of this contaminant advected through a unit area in a unit time may be determined simply as

$$A_t = \bar{V}c \tag{7}$$

where A is the transport rate of the characteristic defined by c and $\bar{V}$ is the mean velocity at the section.

Within a particular fluid element such as one of the channel segments defined for the model the change in the characteristic, c, which is brought about by advection may be determined by the net transport of material through the boundaries of the element. The rate of change in concentration within the element itself is dependent upon the concentration gradient along the channel in which the element is located. This relationship is expressed by

$$\frac{dc}{dt} = \bar{V} \frac{dc}{dx} \tag{8}$$

In Figure 7 the advective transport process defined by equations 7 and 8 is illustrated graphically. It can be seen in the lower sketch that the rate of pollutant advection, A, through a unit interface is a function only of velocity and concentration and that this process leads to a progressive change in the concentration, both in time and space. The gradual modification of concentration at successive times is illustrated by the center sketch. The gradient, dc/dx, is also noted to change with time and space as is evident by comparison of the center and upper sketches. Changes in concentration with time are defined in terms of this gradient in equation 8.

It will be appreciated that the mean velocity, $\bar{V}$, in the real Delta system is also a function of space and time and therefore, a direct solution of either of equations 7 or 8 is possible only for the simplest of cases. Because of this fact and due to the general complexity of the system it will be essential to redefine advective transfer in finite difference form. It is appropriate first, however, to examine the other important transport process, eddy diffusion, and then to set up both formulations for numerical solution.

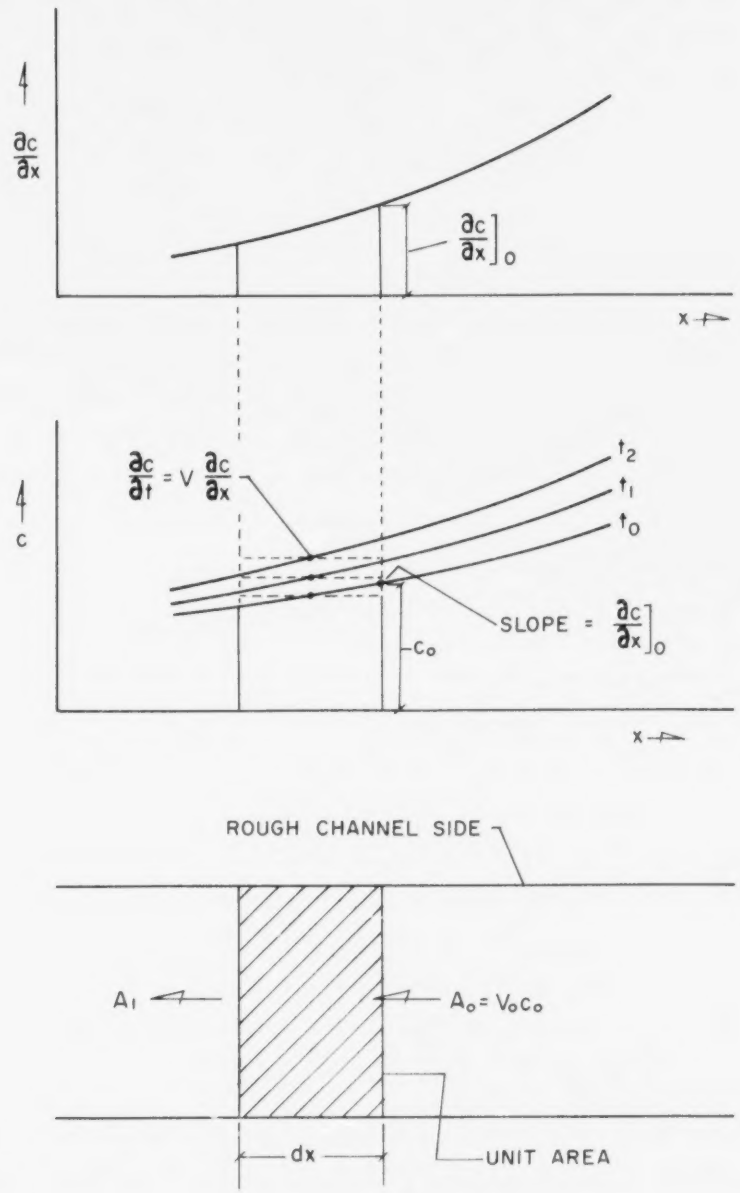


FIGURE - 7
TRANSPORT BY ADVECTION

Eddy Diffusion

The term "eddy-diffusion" for the case of fluid motion is used to describe that process involving the exchange of properties between two adjacent fluid bodies. While the process which is of concern in the present instance is not truly one of diffusion in the classical sense, a strong analogy exists between the behavior of fluid masses, or "eddies", and the nature of molecular motion. This analogy has encouraged fluid dynamicists to adapt to their special problems many of the solutions which have been developed for application in gas dynamics, heat transfer, and chemistry. In fact, these fields have already contributed enormously toward a better understanding of a perplexing dynamic problem, turbulent flow. This improved knowledge serves well the requirements of the investigation reported herein.

It seems unnecessary at this juncture to derive completely from basic considerations the necessary equations for eddy diffusion. It may suffice for present purposes to state that the amount of material which may be diffused through a unit area in a unit time in a turbulent regime is a function of the concentration gradient, $\partial c/\partial x$, and a coefficient, K_d, which may be characteristic of a particular fluid environment.

This relationship may be stated as

$$D_t = K_d \frac{\partial c}{\partial x} \qquad (9)$$

where D_t is the rate of transport of pollutant by the eddy diffusion mechanism. This equation is a statement of what is popularly known as "Fick's First Law".

The application of the Fickian diffusion analogy to the fluid environment in a Delta channel is illustrated in Figure 8. It will be noted that the rate of transport by diffusion of a pollutant, defined by the concentration c, within a channel element of length, dx, at any particular time is proportional to the concentration gradient, $\partial c/\partial x$, and the coefficient, K_d. Both these terms are variable in time and space, the later being closely related to the advection velocity of equations 7 and 8.

The center sketch of Figure 8 illustrates the probable change in concentration with distance along the channel and with time as a result of eddy diffusion alone. It will be noted, carrying forward the same line of development as for advective transport,

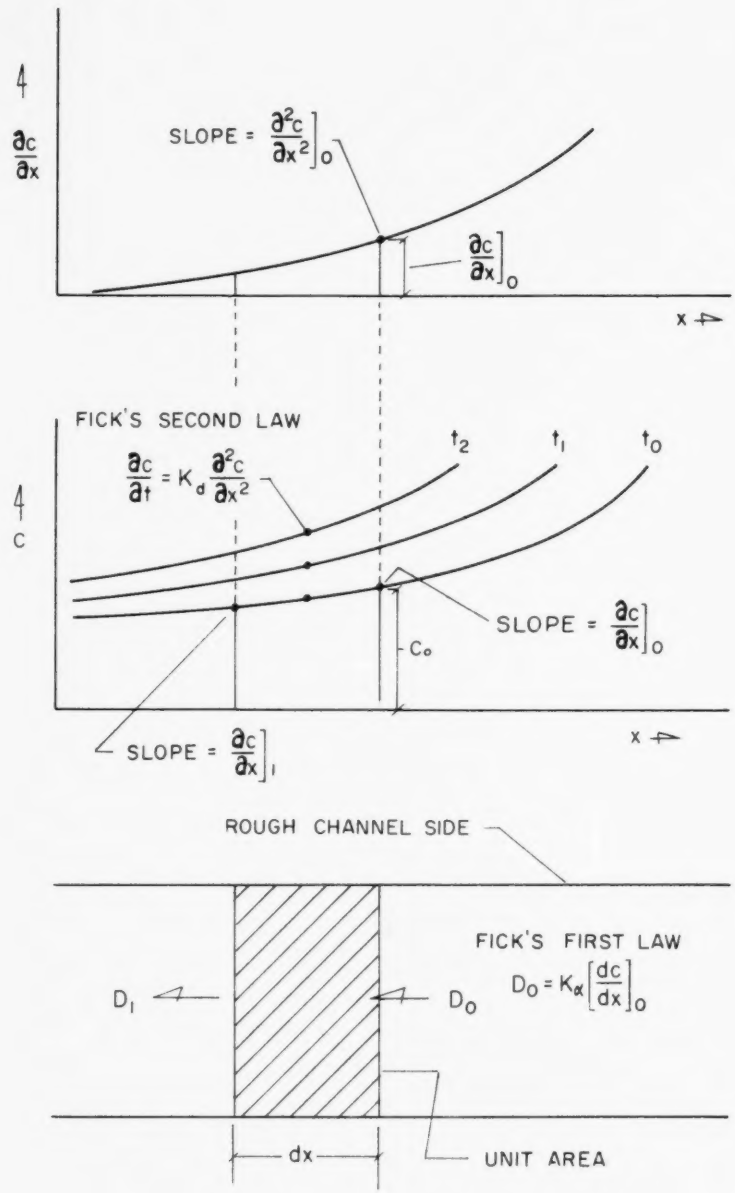


FIGURE-8
TRANSPORT BY DIFFUSION

31a

that the rate of change of concentration with respect to time, $\partial c/\partial t$, is dependent on the second derivative of c with respect to x. This relationship, known as "Fick's Second Law" is

$$\frac{\partial c}{\partial t} = K_d \frac{\partial^2 c}{\partial x^2} \qquad (10)$$

K_d, as will be explained shortly, is highly variable. It depends both on the physical dimensions of the system in which eddy diffusion takes place and on the hydrodynamics of the environment.

Advection and Eddy Diffusion Combined

It will be noted that the net change in amount of material within a channel element over any particular time interval is equal to the net change by both processes of advection and diffusion. The former process, it will be noted, is independent of the concentration gradient while in the latter case a direct proportionality between diffused transport and concentration gradient indicates that this type of transport could occur against advected flow. As a matter of fact, in a flowing system it may be expected that an equilibrium will be reached between these two processes such that the net transport will be zero. This relationship may be expressed by the equation

$$\bar{V} \frac{\partial c}{\partial x} - K_d \frac{\partial^2 c}{\partial x^2} = 0 \qquad (11)$$

When advection and eddy diffusion are combined to bring about a change in concentration gradient there results

$$\frac{\partial c}{\partial t} = \bar{V} \frac{\partial c}{\partial x} + K_d \frac{\partial^2 c}{\partial x^2} \qquad (12)$$

This equation is in the classical form used in solution of estuarial water quality problems where it has been possible to measure directly the concentration as functions of distance and time. The assumptions usually made to obtain a solution are 1) that the estuary is completely mixed vertically, 2) that the flow and

diffusional transport are unidirectional, i.e. one dimensional flow prevails, and 3) that $\bar{V}$ and K_d are constant.

In dealing with a discretized system such as that shown in Figure 4 it is more convenient to use the "First Law" expressions for the rate of mass transport by the combined processes of advection and diffusion. The total transport then becomes

$$M_t = A_t + D_t = \bar{V}c + K_d \frac{\partial c}{\partial x} \qquad (13)$$

where M is the total transport per unit area per unit time for a point at which $\bar{V}$ and K_d are given. In order to effect a numerical solution in which $\bar{V}$ and K_d are functions of x and t Equation 13 may be rewritten in finite difference form. Including appropriate physical constants the transport equation becomes

$$\Delta C = \Delta C_a + \Delta C_d = \left(A_i \bar{V}_i c_j + K_d A_i \frac{\Delta c}{\bar{X}_i} \right) \Delta t \qquad (14)$$

where for a particular channel reach

C	=	total weight of pollutant transferred, pounds
t	=	time, seconds
A	=	channel cross section, square feet
$\bar{V}$	=	mean channel velocity, feet per second
c	=	concentration of pollutant, pounds per cubic foot
K_d	=	eddy diffusion coefficient, square feet per second
X	=	length of channel reach in feet

Diffusion vs Dispersion

It is apparent in consideration of Equation 12 with respect to the physical problem that the coefficient of eddy diffusion is not a constant but is a function of time and space. This is clearly the case if one appreciates the dependence of K_d on fluid turbulence and channel geometry. For a one-dimensional channel

$$K_d = f(x, t) \qquad (15)$$

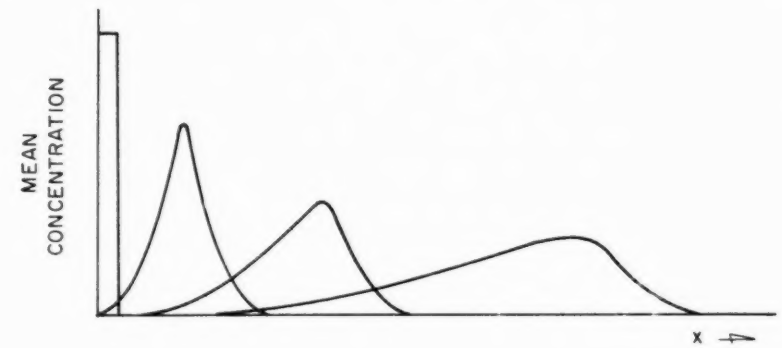
i.e., K_d depends on the nature of the channel section at the location under consideration

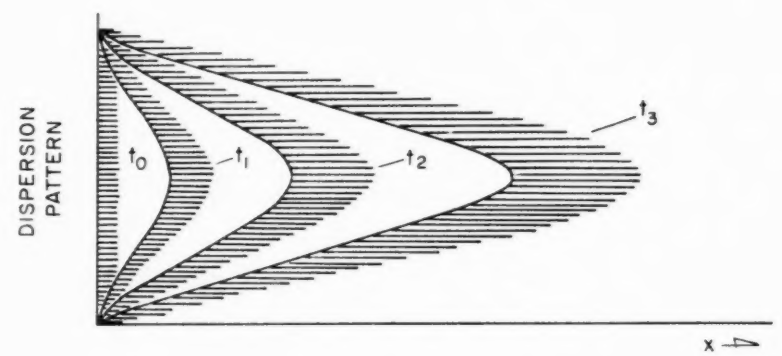
and the state of flow at the particular time being considered.

Equation 12 is often used as a means for deriving K_d, a procedure which can lead to erroneous conclusions concerning the nature of the eddy diffusion process. Since it is convenient to measure or estimate $\overline{V}$, the mean velocity of flow, and to determine the variations of concentration with time and distance it is possible to compute an "effective" diffusion coefficient. This coefficient may be one which satisfies the equation for the specific case or cases studied but may be wholly invalid for other conditions or locations. This may be better appreciated if K_d determined in this manner is looked on as reflecting the combined effect of two different processes, diffusion and dispersion.

Dispersion is the mechanism through which fluid particles undergo relative displacement solely by virtue of differences in mean velicities along two adjacent stream lines. This process is illustrated in Figure 9 where it can be seen that fluid particles in the central portion of a channel are transported rapidly while those close to the rough banks are greatly retarded. If one could label a transverse lamina with a detectable material and sample across the channel at various times and distances from the referrence point a distributed pattern of concentration would result. This pattern would not be "normal" as suggested by a constant K_d but would have a peak shifted to the right with the main body of the flow and a long tail to the left representing retarded flow.

This indicated behavior is distinct from that induced by statistical eddy motion or "eddy diffusion" in which properties of the flow are exchanged by transfers of momentum related to variations in the mean flow which are statistically homogeneous throughout the cross section and along streamlines. For this reason Equation 12 is only valid in the rigorous sense if $\overline{V}$ is invariant for all points in the section taken transverse to the x-axis. What is required to correct for this failing is a means of representing functionally the variations of $\overline{V}$ from point to point in the section so that advective transport properly includes the dispersion phenomenon. Mathematically this modification is difficult — virtually impossible for the practical case in point. However, as will be illustrated shortly, there are some characteristics of the numerical solution which facilitate reasonable approximations of the dispersion effect.





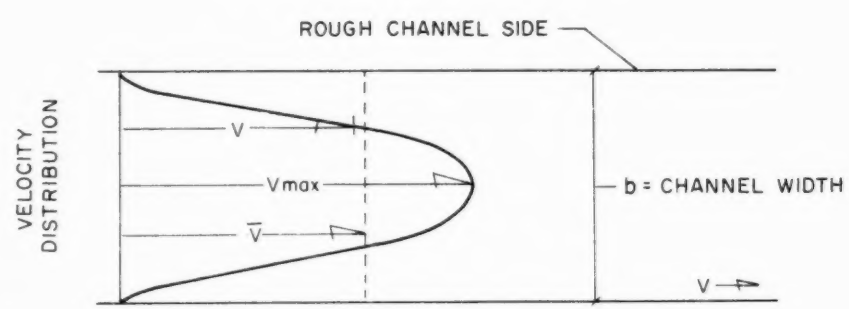


FIGURE - 9
ADVECTIVE DISPERSION

It is all too obvious at this point that the variable nature of K_d as suggested in Equation 15 precludes direct solutions of Equation 12 except for the simplest of systems. Certainly, to seek a direct solution for the Delta network would be unthinkable. Nevertheless, numerical techniques applied through Equation 14 should permit the stepwise approximation of the function. In the numerical method lies the only really practical entry to the problem. It remains to be shown how K_d can be related to the hydraulic and physical properties of the channel.

Eddy Diffusion, Scale, and Energy Dissipation

On the basis of previous experience with eddy diffusion in channels and consistant with the theory of Kolmogoroff it can be stated that

$$K_d = f(\text{rate of energy dissipation, scale of the phenomenon})$$

From laboratory experiments and field measurements it has been demonstrated (1) that

$$K_d = C_1 E^{1/3} L_e^{4/3} \qquad (16)$$

where E is the rate of energy dissipation per unit mass, L_e is the statistical mean size of eddies participating in the mixing process, and C_1 is a complicated function of relative channel roughness (2). For practical purposes C_1 may be taken as constant over the range of roughnesses encountered in the Delta channel. E may be determined by

$$E = \bar{V} g S \qquad (17)$$

(1) Orlob, G. T., Eddy Diffusion in Open Channel Flow, Contribution No. 19, Water Resources Center, University of California, November 1958.

(2) Orlob, G. T., "Eddy Diffusion in Homogeneous Turbulence" Trans ASCE v 126, 1961, pp 397-438

where S is the slope of the energy grade line = $\partial H/\partial X$. The mean eddy size may be taken as proportional to some controlling channel dimension, perhaps the width or depth, or even the hydraulic radius.

Adopting the channel depth, y, as a measure of scale and making substitutions for E and L_e there results

$$K_d = C_2 \bar{V} y^{8/9} \tag{18}$$

where

$$C_2 = \frac{C_1 g^{1/3} n^{2/3}}{(1.486)^{2/3}} \tag{19}$$

For computational purposes it is convenient to simplify (19) to the form

$$K_d = C_3 \left| \bar{V} \right| y \tag{20}$$

which can be used to compute the eddy diffusion coefficient for each channel segment for which n is known and for each time interval over which $\bar{V}$ and y are held constant. The absolute value signs are included to indicate that the transport by eddy diffusion is independent of the sense of flow and depends only on the sign of the concentration gradient as given in (14).

Preliminary studies of dye dispersion in the lower Delta channels conducted by the California Department of Water Resources indicated that C_3 would have to be somewhere within the range of 1 to 10. In one set of observations the "effective" diffusion coefficient was about 110 (C_3 = 4.2). These values were recognized, however, as including the effect of advective dispersion as well as that of eddy diffusion and should be accordingly reduced to represent only the latter effect. For preliminary studies with the model C_3 was taken as 0.042, which from equation 20 would give values of K_d ranging to about 5 for the larger channels of the system flowing at maximum velocity.

DEGRADATION AND PURIFICATION

Non-conservative pollutants, such as organic material added through sewage discharge, are converted by certain processes to constituents which are stable, i.e. "conservative", or they are extracted from the environment. That is to say, from the point of view of their measurable concentration in the system, they are "degraded". Other constituents, such as oxygen supplied by natural reaeration, may be consumed in the degradation process. The two processes are therefore closely linked, the fate of one constituent determining the concentration of the other.

Because of their special significance in relation to Delta water use it is especially important that the water quality model have the capability of representing biochemical degradation of organic matter and its associated demand on oxygen resources. The theoretical and practical considerations related to their inclusion in the model are treated briefly in the following section.

Degradation - "Decay"

The biochemical stabilization of organic material is assumed to proceed at a rate directly proportional to the amount of unstabilized material remaining, such that

$$\frac{dL}{dt} = -K_1 L \qquad (21)$$

where L is the biochemical oxygen demand (BOD) at time t, and K_1 is a reaction velocity constant.

Equation 21 is that of the classic "unimolecular" reaction which has been so universally applied to describe the process of organic stabilization in nature. It is convenient for its simplicity and, even though it does not adequately describe the more intricate phenomena of stabilization, it is a practical formulation for use in the present model.

Most of the improvements which have been suggested for equation 21 have been directed to a better definition of K_1 although some attention has been given to separation of L into its components, each of which might be stabilized in some

unique way. Further, there has been some effort expended to consider temporal variations in both K_1 and L, the most notable of which is probably the work of Frankel (3).

It is acknowledged that these refinements may ultimately be important in the study of water quality. In fact, the model is equipped to accomodate variable K_1 and L, both spacially and temporally. For the initial studies, however, the traditional assumptions were used. The reaction velocity was taken constant throughout the system at a rate of about 20 percent per day and both K_1 and L were taken as invariant with time.

For practical solution in the model Equation 21 can be written in finite difference form for a junction as

$$\Delta C_b = -K_{1j} L \nabla_j \Delta t \qquad (22)$$

where ΔC_b is the total decay in pounds during time, Δt. In this form and with K_1 and the initial value of L known it is possible to describe the history of L since the tidal channels experience their characteristic oscillation and the processes of advection and diffusion are active.

Reaeration

The process of reaeration or oxygen transfer, which supplies part of that which may be required in stabilization of organic material may be stated in the familiar form

$$\frac{d(c_s - c)}{dt} = -K_2 (c_s - c) \qquad (23)$$

where c is the concentration of dissolved oxygen in water, c_s is the saturation concentration at equilibrium with oxygen in the atmosphere under the specified

(3) R. J. Frankel "Water Quality Management: An Engineering-Economic Model for Domestic Waste Disposal" Ph.D. Thesis, University of California 1964

environmental conditions, and K_2 is a transfer coefficient or reaction velocity constant. In the form most suited to numerical calculation this relation may be rewritten

$$\Delta C_o = K_{2j} (c_s - c_j) V_j \Delta t \qquad (24)$$

where ΔC_o is the total weight of oxygen transferred in time Δt. Similarly, as for decay, if c_s, K_2, and the initial value of c are known it is possible to determine, for a succession of intervals, the history of c.

Each of the quantities presented in equation 24 can be assigned, or calculated as functions of other characteristics of the Delta system. Of particular interest, however, because of its more complicated variability is the coefficient K_2. It is perhaps appropriate at this point to explore briefly the methods available for the election or computation of K_2 and to outline some of the difficulties which might be encountered.

The Reaeration Coefficient

The reaeration coefficient is dependent on a variety of environmental conditions. In the traditional Streeter-Phelps formulation (4) of the oxygen sag K_2 is treated as a constant for a particular reach of stream but was made a function of temperature through an empirical equation of the form

$$(K_2)_T = (K_2)_{20} \theta_K^{T-20} \qquad (25)$$

where θ_K is a characteristic of the oxygenation rate, and T is the Temperature of the water. For T in degrees Centigrade, θ_K has been observed to have a range from 1.015 to 1.047.

Observations, both in the laboratory and in the field, have confirmed the dependence of K_2 on interfacial area and depth of water. However, in the absence of a more complete understanding of the mixing of fluid masses, which brings

(4) Streeter, H. W. and E. B. Phelps, "A Study of the Pollution and Natural Purification of the Ohio River" U. S. Public Health Bull. 146 (1925)

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oxygen deficient water to the interface and transfers saturated water to greater depths it has been the custom to characterize streams only in a crude physical way. Such qualitative descriptions as "turbulent","sluggish", "ponds and backwaters", etc. have been attached to environmental situations in which K_2 has ranged through several orders of magnitude, say from 0.1 to 10. Often, for illustrative purposes, a value of 0.2 has been taken as typical of "slow moving" streams (velocities less than 2 feet per second) such as might typify the Delta network.

Recently, there have been some significant investigations aimed at developing better expressions for K_2 which recognize the importance of fluid turbulence in the reaeration process. Among these the most notable are the studies of O'Connor and Dobbins (5), Churchill, et al (6), Krenkel (7) and Dobbins (8), each of whom adopted a slightly different approach to the problem.

O'Connor and Dobbins offered an equation which recognized the fluid motion, depth of fluid relative to the interface, and molecular diffusivity as important parameters. In the symbology used herein the equation* is

$$K_2 = \frac{(D_M \bar{V})^{1/2}}{y^{3/2}} \qquad (26)$$

(5) O'Connor, D. J. and Dobbins, W. E., "Mechanism of Reaeration in Natural Streams", Trans ASCE, v 123, 1958.

(6) Churchill, M. A., Elmore, H. L., and Buckingham, R. A., "The Prediction of Stream Reaeration Rates", J. San Engrg. Div. ASCE v 88 July 1962

(7) Krenkel, P. A., "Turbulent Diffusion and the Kinetics of Oxygen Absorption", Ph.D. Thesis, University of California, Berkeley

(8) Dobbins, W. E., "BOD and Oxygen Relationships in Streams", J. San Engrs. Div. ASCE June 1964

* $K_2 = 2.303 k_2$ where k_2 is the corresponding coefficient for the integrated form of equation 23 with the base 10 instead of the base e.

where D_M is the molecular diffusivity of oxygen in water in square feet per day, $\bar{V}$ is the mean velocity in feet per day, and y is the depth in feet. D_M has a value of 0.00207 square feet per day at 20° C. and can be estimated for other temperatures by the equation

$$D_M = D_{M_{20}} (1.037)^{T-20} \tag{27}$$

Churchill, et al in statistical correlation of many stream reaeration parameters developed the multiple correlation equation

$$K_2 = 11.6 \frac{\bar{V}^{0.969}}{y^{1.673}} (1.024)^{T-20} \tag{28}$$

in which T is the temperature in degrees Centigrade, $\bar{V}$ is the velocity in feet per second, and y is the depth in feet. Churchill suggested a simplification which reduces Equation 28 to

$$K_2 = 11.5 \frac{\bar{V}}{y^{5/3}} \tag{29}$$

Krenkel sought to relate the reaeration coefficient to turbulence as described by the eddy diffusion analogy. His investigation led to a proposal that

$$K_2 = \text{Constant} \frac{K_d}{y^2} \tag{30}$$

in which K_d is the eddy diffusion coefficient corresponding to the direction of advection. Using 58 sets of experimental data collected from a laboratory flume operated under controlled conditions Krenkel obtained a multiple correlation equation

$$K_2 = 15.9 \; K_d^{1.150} \; y^{-1.915} \tag{31}$$

which tends to support the functional form proposed. In this equation K_d is in units of square feet per second, y is in feet and K_2 is in units of days^{-1}.

It should be noted, however, that K_d for these experiments was the "effective" diffusion coefficient for a channel of uniform section in which the advective dispersion effect was minimized by using shallow depths, a rough bottom, and smooth sides. In this case the assumption was the "effective" diffusion and eddy diffusion were practically identical and that advective mixing was negligible.

It will be noted that combining equation 30 with equation 20 results in the relation

$$K_2 = \text{Constant} \frac{\bar{V}}{y} \tag{32}$$

A plot of the parameter $\bar{V}/y$ against the experimental data obtained under controlled laboratory conditions confirms this equation as a satisfactory tool for prediction of the reaeration coefficient. From this analysis and comparison with equations 26 and 29 a constant of 2.5 is suggested as appropriate for application to prototype streams.

Dobbins proposed a somewhat complicated model of the aeration process which may be stated as

$$K_2 = \frac{C_A K_L}{y} \tag{33}$$

where C_A is a coefficient used to correct the apparent surface area to that which would result from surface distrubances, wind, etc. and K_L, the absorption coefficient is taken as

$$K_L = (D_M r)^{1/2} \coth \left(\frac{r \delta^2}{D_M} \right) \tag{34}$$

In this latter equation r is the frequency of replacement at the interfacial surface of a liquid film which has a thickness, δ. Dobbins invokes the Kolmogoroff similarity principle as a means of relating rate of film renewal and thickness. The essential equations necessary to determine K_2 also include the following:

$$\delta = C_4 \left(\frac{\nu^3}{E} \right)^{1/4} \tag{35}$$

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$$C_4 = 0.65 + 15{,}000\left[\left(\frac{\nu^3}{\frac{E}{y^2}}\right)^{1/4}\right] \qquad (36)$$

$$r = 14.3\,\frac{\rho\,\nu^3}{\sigma\,\delta^3} \qquad (37)$$

where terms as yet undefined are as follows:

ν	=	kinematic viscosity
E	=	rate of energy dissipation (Equation 17)
ρ	=	fluid density
σ	=	surface tension

The coefficient, C_4, actually ranged from about 0.3 to 1.2 in the region where data from natural streams was used to develop the equation. The second member of Equation 36 represents data from laboratory experiments reported by Krenkel and Orlob (9).

In summarizing his theory Dobbins acknowledges the dependence of his equations on empirical constants such as C_4 and C_A as well as the constant in Equation 37 However, he states that the effects of varying these constants are not great as long as consistent values are used.

From the practical viewpoint of implementing study of the impact of organic wastes on the Delta system it appears that either of Equations 26, 29 or 32 could be applied to the model. The equations of Dobbins do not lend themselves well to the practical problem as it must be handled in the model by numerical computation. Table 3 below illustrates the range of predicted values of K_2

(9) Krenkel, P. A. and G. T. Orlob "Turbulent Diffusion and the Reaeration Coefficient" J. San. Engr. Div. ASCE V88 March 1962

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which might be expected for large and small Delta channels under extremes of tidal motion.

TABLE III RANGE OF EXPECTED REAERATION COEFFICIENTS

		Eq. 26 O'Connor - Dobbins	Eq. 29 Churchill et	Eq. 32 Krenkel - Orlob
Deep Channel:				
y = 40 ft., $\bar{V}$ =	1 fps	0.051	0.024	0.063
	2 fps	0.072	0.048	0.125
	3 fps	0.088	0.072	0.188
	4 fps	0.102	0.096	0.250
Shallow Channel:				
y = 5 ft, $\bar{V}$ =	1 fps	0.407	0.249	0.250
	2 fps	0.577	0.498	0.500
	3 fps	0.706	0.747	0.750
	4 fps	0.814	0.996	1.000

Because of the weight of statistical data utilized by Churchill it must be accepted that Equation 29 is reasonably reliable for the shallower channels. However, some question might be raised concerning the deeper channels, particularly those that sustain a high degree of turbulence due to tidal oscillation. In this latter case, perhaps, O'Connor's relation might be expected to serve better. Equation 32 seems to be a good compromise, agreeing well with the Churchill relation for the shallower channels and offering reasonable values for the deeper sections.

In summary, it should be stated that none of the theories or prediction equations offered is completely satisfactory for all conditions. None, for example, has been derived from a study of estuarial mixing and turbulence. It remains to be determined which of these, or what new and more suitable relation, should be used for the peculiar problems of the Delta system. For the present the simplest of equations is perhaps the best.

It must also be recognized, at this point, that no effort has yet been made to introduce biological factors which affect the oxygen balance in the estuary. This does not mean that the present model is not correct in principle, only that it may need additional elements. Photosynthetic production of oxygen by algae, for example, is of acknowledged importance and must ultimately be incorporated. At the present time, however, too little is known of this phenomenon to permit any more than an arbitrary empirical approach to the problem. It will be an important area for future study in which the present model will serve as a useful tool.

IV WATER QUALITY MODELS

GENERAL

During the course of the investigation reported herein mathematical representation of the processes affecting the quality of Delta water evolved through a series of progressively more complicated forms. Initially, using information provided by the dynamic flow program, a simple steady-state model for a conservative pollutant was developed and applied. Subsequently, a dynamic steady-state model for advective transport of conservative pollutants was constructed, diffusional processes were included, and the model was tested. Finally, the processes essential to depicting the fate of non-conservative materials were incorporated in the model to bring it to its present form. In this state it is capable of application to a wide variety of water quality problems and should serve as a useful tool in evaluation of the many alternatives for construction and operation of Delta water transfer facilities.

Because each form of water quality model studied has certain utility for various classes of problems it is desirable that their capabilities and limitations be appreciated. The following discussions present briefly the characteristics of the steady-state model useful for salt routing studies and then develop more fully the structure of the dynamic water quality model with all of its features.

STEADY-STATE WATER QUALITY MODEL

For any system of interconnected channels which have attained a state of stable equilibrium hydraulically it is possible to write at any junction that

$$\Sigma Q_i = 0 \qquad (38)$$

where Q_i represents any flow entering or leaving the junction in question. In Equation 38, which is the equation of continuity, it is implicit that flows are steady and there is no change in storage within the system or any component of the system. This condition, of course, cannot be satisfied exactly at a specified time for the Delta network because of tidal influences. Nevertheless, if long-term means are deter-

mined for each element of the system under conditions of steady input it is possible to satisfy the equation and to use the resulting flows for an informative study of pollutant transport through the system.

Using the WRE Dynamic Flow Program values of discharge in each tidal channel can be integrated over one full tidal cycle and net flows can be determined by

$$\bar{Q}_i = \frac{\int_{t_1}^{t_2} Q_1 \, dt}{t_2 - t_1} \qquad (39)$$

These values may then be confirmed as satisfying Equation 38 within acceptable limits of accuracy and a general pollutant balance equation can be written for each junction in the system as follows: The known flows in channels connected to the junction are designated Q_i ($i = 1 \ldots n$). Consumptive use and return flow are identified respectively as Q_u and Q_a. Concentration of return flow, c_a, is assumed to be a known quantity while inflowing water arrives at concentration, c_i, and outflowing water leaves at concentration, c_j. The resulting conservation equation for a typical junction then becomes:

$$c_1 Q_1 + c_2 Q_2 + c_3 Q_3 + \cdots + c_j Q_n + c_j Q_u + c_a Q_a = 0 \qquad (40)$$

Similar equations are written for each junction in the system and finally solved simultaneously for the unknown concentrations.

The computer program for solution of these equations was developed for use on the IBM 7094 computer, but can be adapted for use on other machines. It has a capacity of 150 junctions and 300 channels. Data input is from cards and therefore can be used with hydraulic solutions obtained from either the transient flow or steady state programs. The data input requires 1) the listing of the total number of junctions and channels in the system; 2) the number of each channel entering a particular junction; 3) the inflows and outflows at each junction; 4) the

contaminant concentration for each inflow; 5) the junction numbers at the ends of each channel; and 6) the flow in each channel.

Results obtained from the salt balance program have been compared with those obtained by the more tedious manual procedures employed by the California Department of Water Resources. Agreement was within a few percent.

It must be acknowledged that results by this rather crude procedure can only be as accurate as the balance of flows obtained in the supporting hydraulic analysis. Moreover, the assumptions of complete mixing at junctions and the failure of this model to represent dispersion and diffusion mechanisms lead to obvious departures from prototype behavior. This is especially notable in the lower Delta where the salt balance program fails to depict salinity incursion simply because net flows as calculated by equation 39 are generally seaward. Nevertheless, qualitatively the model has proven itself to be a useful tool which can be inexpensively applied for purposes of preliminary evaluation of project alternatives. It may be regarded as utilitarian detour along the path toward a more rigorous mathematical model of the water quality environment.

DYNAMIC WATER QUALITY MODEL

General Structure

The processes defining circulation and exchange of pollutants are obviously dependent on the hydraulic behavior of the system. In the case of the Delta models this consideration dictated that either hydraulics and water quality be treated simultaneously by the same computer program, or that they be handled by separate but compatible programs. The sheer complexity of the system to be solved precludes the practical conjunctive operation of models of different type, i.e. analog plus digital or physical plus digital.

It is evident on review of the problems associated with wave propagation through the system that much less detail is required for characterization of water quality changes and that a common program offers no real advantage over separate programs with compatible structure. Where 2.5-minute integration intervals were

needed for the hydraulic solution it was anticipated that nothing less than 30 minutes of prototype time would be needed for water quality studies. In the latter case, also it might be desirable to follow the phenomena for more than 30 days while in the former two tidal cycles (about 25 hours) was sufficient to give representative behavior.

The water quality program, therefore, was developed with the same basic structure as the dynamic flow program but was made separate, accepting as basic input a complete time-history of hydraulic behavior previously recorded on magnetic tape. This tape contained the value of the head at each junction and the flow and cross-sectional area for channel at the end of every time interval. The same information may be supplied to any number of special water quality cases to be examined and may be repeated ad infinitum within one particular study.

Finite Difference Formulation

It is appropriate to summarize at this point the several basic phenomena which are considered by the dynamic water quality program and to state in finite difference form the appropriate computation equations. They are as follows:

a. Advection (and Advective Dispersion)

$$\Delta C_a = A_i \bar{V}_j c_j \Delta t \tag{41}$$

b. Eddy Diffusion

$$\Delta C_d = K_d A_i \frac{\Delta c}{X_i} \Delta t \tag{42}$$

c. Degradation-Decay

$$\Delta C_b = K_{1j} L \bar{V}_j \Delta t \tag{43}$$

d. Reaeration

$$\Delta C_o = K_{2j} (c_s - c_j) \bar{V}_j \Delta t \tag{44}$$

e. Import-Export

$$\Delta C_e = Q_e c_e \Delta t \tag{45}$$

In this set of equations, concentrations, as represented by c, c_s, c_e, c_j and L, are all given in pounds per cubic foot and ΔC's would be calculated in pounds. In the numerical operation concentrations can be specified in ppm such that

$$c\ (\text{ppm}) = \frac{c\ (\text{pounds per cu. ft.})}{\gamma} \times 10^6 \tag{46}$$

where γ is the specific weight of water.

It will be noted that advection, eddy diffusion and import-export are operative for either conservative or non-conservative pollutants while for some special cases of non-conservative pollution all processes are operative. For example, to produce the "oxygen sag" under dynamic conditions the "load" would be supplied to the system as (e), it would decay as (c), oxygen would be utilized equivalent to (c) and supplied to the system in accord with (d) and finally, both the organic load and the oxygen resource would be transported by the process of (a) and (b).

Water removed from the system by (e) may be either pure or polluted provided the relationship

$$Q_j' + Q_j'' + Q_j''' = Q_j \tag{47}$$

is observed where

Q_j' = pure water removed, $c = 0$

Q_j'' = water removed at concentration c_j

Q_j''' = water added to system at any specified concentration and

Q_j = advected flow from hydraulics program.

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Special Problems in Application

Starting conditions and stability: Just as in the case of the hydraulic program a complete set of initial values must be given for the system. During the early stages of operation the system will function in a transient state, tending to approach a dynamic steady state in which histories of concentration change tend to repeat themselves. The time required to reach this state is highly variable depending on the process involved and on the assumed starting conditions.

Diffusional movement of material, for example, occurs relatively slowly when contrasted to advective translation and dispersion. In fact, it may well be that the time required to establish a "dynamic steady state" for this process is much longer than the period over which the hydrologic conditions at the boundaries of the model may be justifiably treated as constant. In certain instances it may be desirable to study the system while it approaches a new equilibrium condition at the hydrologic situation of some future date. In these cases diffusional movement may prove to be of important consequence. Additional investigation will be required both with the model and in the field to establish the real significance and relative importance of the various mixing and exchange processes which control pollutant levels in the Delta system.

It is desirable whenever possible to start the model close to an equilibrium solution. Hence, there is considerable need for experience with the prototype. Preliminary studies with the model suggest that good convergence can be obtained for most cases with conservative contaminants in about one week of prototype time. Non-conservative pollution problems may require about two weeks of simulation. Although these periods of convergence may be of special interest for certain problems they will probably not provide the extreme values which are of greatest interest in pollution studies.

Stability of the solution is not so difficult to control as for the hydraulic program. There are, however, two possible sources of difficulty. First, the flow advected from one junction to another in a given interval, Δt, should not be greater than the volume of the connecting channel. Since the volume of available water at a junction is equal only to the half-volumes of connecting channels such an occurrance could lead to the anomalous situation of transferring more pollutant out of the junction than there was initially available.

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Second, inordinately high diffusion coefficients may also result in transfers of pollutant, particularly near starting conditions, which may exceed the capacity of the storage at junctions. This condition, like that associated with advection, may be corrected by shortening the interval, Δt.

Induced Advective Dispersion: It was noted previously in the section of this report dealing with "Diffusion vs. Dispersion" that the latter phenomenon is difficult to represent faithfully in the mathematical model. This is principally due to lack of knowledge of prototype behavior, i.e. the variations in advective velocity from point to point in the channel cross-section and at different times.

One approach to this dilemma which in many investigations has been inadvertently adopted is to represent this process as a part of eddy diffusion, simply adopting an appropriate coefficient, K_d. However, this method fails to recognize the real phenomenon as being associated with advective transport.

A second approach is to modify the advective transport term so that some material is permitted to move ahead of the mean position of the advected "front" and some is retarded, in much the same manner as occurs in the prototype due to variations from the mean stream velocity. This might be accomplished computationally by increasing the number of elements representing a channel; using several parallel systems with differing resistance properties. These could be recombined at junctions to form the composite effect desired. However, the impracticality of this attack on the problem with a system already as complex as that of the Delta is very evident.

A variation on this second approach, which occurs coincidentally due to the numerical procedure employed in the model and because of the influence of tides on channel velocity, affords a more practical means of representing the advective dispersion effect. Because the Delta system consists of discrete channel segments of fixed length within which the velocity is continually changing, the advective travel of a pollutant is almost always substantially different from the channel segment length usually it is less if the volume criterion cited in the previous section is adhered to. Since, it is assumed computationally that complete mixing takes place at the end of

each interval, Δt, it is apparent that some part of the pollutant will appear to move ahead more rapidly than the mean flow. The total amount in motion does not change, only the distribution. The distribution which results from this computational phenomenon is a function of the ratio, $\bar{V}\Delta t/X$, where $\bar{V}$ is the mean velocity during the interval, Δt, in a channel segment of length, X.

It is recognized that the effect produced results from a computational improvisation, but also that it has certain properties that are in accord with the real behavior of the prototype. Among these are:

1. No transport occurs except in the direction of flow, i.e. no pollutant can be moved beyond a point in the estuary where no current reversal is experienced.

2. The computational correction, dependent on the ratio, $\bar{V}\Delta t/X$, is greatest in those channels where advective dispersion would be most important, i.e. wide, relatively shallow channels of the lower Delta. It occurs where it is most needed.

3. Transport by advective dispersion, as it is represented in the model by this computational mechanism, tends to reduce eddy diffusion transport by decreasing the concentration gradient. The combined effect of all transport processes, advection, advective dispersion, and eddy diffusion is still represented in the model.

There is an apparent need for additional investigation of the advective mixing process, particularly as it is related to and distinguished from eddy diffusion. It is important to meet this need through carefully planned field studies of transport mechanisms. With the aid of reliable data on the prototype the model will be able to serve more completely its role as a practical engineering tool.

Dynamic Water Quality Program

Figure 10 presents the basic computational program written in FORTRAN IV for the IBM 7094 computer. While the various computational processes are identified for purposes of clarity the symbology used is that of the digital coding language. The reader unfamiliar with this language is advised to review any of the many excellent published manuals on FORTRAN programming.

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```
DYNQAL   DYNAMIC WATER QUALITY PROGRAM      WATER RESOURCES ENGINEERS


      DIMENSION ALPHA(48),Y(750),YT(750),QIN(750),NCHAN(750,6),
     *CLEN(1000),B(1000),AREA(1000),AREAT(1000),CN(1000),V(1000),
     *VT(1000),Q(1000),R(1000),AK(1000),NJUNC(1000,2),YSPEC(1000),
     *CON1(1000),CON3(1000),DVLEN(4000),TSUMA(750),TSUMA2(750),JPRT(100)
      EQUIVALENCE (AK,CN),(CCN1,R),(TSUMA,B),(TSUMA2,CLEN)
      COMMON ALPHA,Y,YT,QIN,NCHAN,CLEN,B,AREA,AREAT,CN,V,VT,Q,R,NJUNC,
     *YSPEC,NJ,NC,NCYC,DELT,DELT2,NEXIT,ICYC,KCYC,SUMQ,NWAVE,SUMC,DVLEN,
     *CON3,L,JPRT

      DIMENSION YNEW(750),C(750),              SALT(750),VOL(750),
     *ASUR(750),CSPEC(750),CIN(50),DIFFK(1000)
      EQUIVALENCE (YNEW,YT),(C    ,DVLEN),(SALT,DVLEN(1000)),(VOL,VT),
     *(ASUR,AREAT),(CSPEC,DVLEN(2000)),(DIFFK,DVLEN(3000))
      DIMENSION COX(750),OSPEC(750),CINOX(50),CXY(750),QNET(1000)
      COMMON COX,OSPEC,CINOX,CXY,QNET

      REWIND 10
      REWIND 3

C**** READ SYSTEM INFORMATION FROM DYNAMIC FLOW PROGRAM

  100 READ TAPE 10,(ALPHA(I),I=1,12),NJ,NC,NCYC,NPRT,DELT,(Y(J),QIN(J),
     *(NCHAN(J,K),K=1,6),J=1,NJ),(CLEN(N),B(N),AREA(N),CN(N),V(N),R(N),
     *(NJUNC(N,I),I=1,2),N=1,NC),NWAVE,(YSPEC(I),I=1,NWAVE),
     *NOPRT,(JPRT(I),I=1,NOPRT)

C**** READ INDEPENDENT CONTROL DATA

      READ INPUT TAPE 5,103,(ALPHA(I),I=25,48)
  103 FORMAT(12A6)
      WRITE OUTPUT TAPE 6,105,(ALPHA(I),I=1,48)
  105 FORMAT(1H1112A6,20X,32H WATER RESOURCES ENGINEERS, INC./
     *       1H 12A6,20X,22H LAFAYETTE, CALIFORNIA/
     *       1H 12A6/1H 12A6////)
  349 READ INPUT TAPE 5,350,NSTART,NSTOP,NQCYC,NQPRT,NODYN,DECAY,CS
  350 FORMAT(5I5,2F10.0)
      WRITE OUTPUT TAPE 6,351,NSTART,NSTOP,NQCYC,NQPRT,NODYN,DECAY,CS
  351 FORMAT(107H *** HYDRAULICS PROGRAM ***          NUMBER OF          PR
     *INT      HYDRAULIC CYCLES PER       DECAY        OXYGEN/
     *       109H START CYCLE      STOP CYCLE      QUALITY CYCLES      INTE
     *RVAL         QUALITY CYCLE          FACTOR     SATURATION//
     *         1H I7,I16,I17,I15,I19,F20.6,F11.2//)
      DO 352 J=1,NJ
      OSPEC(J)=CS
  352 COX(J)=CS
      IF(SENSE SWITCH 6)96,98
   96 WRITE OUTPUT TAPE 6,97
   97 FORMAT(54HOTHI  RUN CONSIDERS ADVECTION AND EDDY DIFFUSION ONLY.)
      GO TO 198
   98 WRITE OUTPUT TAPE 6,99
   99 FORMAT(86HOTHI  RUN CONSIDERS ADVECTION, EDDY DIFFUSION, DECAY, OX
     *YGEN DEPLETION AND REAERATION.)

C**** MAKE EXTRACT OF HYDRAULICS TAPE
C**** COMPUTE NET FLOWS

  195 IF(SENSE SWITCH 5)225,159
  199 NRITE=NSTART
  200 READ TAPE 10,ICYCTF,(YNEW(J),J=1,NJ),(V(N),Q(N),N=1,NC)
      IF(ICYCTF-NSTART)200,210,210
  204 IF(ICYCTF-(NSTART-NODYN))200,205,207
  205 DO 206 N=1,NC
  206 QNET(N)=0.5*Q(N)
      GO TO 200
  207 DO 208 N=1,NC
  208 QNET(N)=QNET(N)+Q(N)
      GO TO 200
  210 DO 211 N=1,NC
  211 QNET(N)=QNET(N)+Q(N)
  214 IF(ICYCTF-NRITE)220,215,215
  215 NRITE=NRITE+NODYN
      WRITE TAPE 3,ICYCTF,(YNEW(J),J=1,NJ),(Q(N),N=1,NC)
  220 IF(ICYCTF-NSTOP)200,223,223
  223 DO 224 N=1,NC
      QNET(N)=QNET(N)-0.5*Q(N)
  224 QNET(N)=QNET(N)/FLOATF(NSTOP-(NSTART-NODYN))
  225 REWIND 3
      REWIND 10

C**** READ QUALITY COEFFICIENTS

      READ INPUT TAPE 5,350,NJREAD
      READ INPUT TAPE 5,355,(J,C(J),CSPEC(J),COX(J),OSPEC(J),
     *I=1,NJREAD)
  355 FORMAT(I5,4F10.0)
      WRITE OUTPUT TAPE 6,356,(J,C(J),CSPEC(J),COX(J),OSPEC(J),Y(J),
     *QIN(J),(NCHAN(J,K),K=1,6),J=1,NJ)
  356 FORMAT(96HOJUNCTION      INITIAL C-FACTOR      INPUT CONC      HEAD
     *    OUTPUT      CHANNELS ENTERING JUNCTION//
     *(1H I5,2F15.0,4F10.2,I8,5I6))
      READ INPUT TAPE 5,350,NSPEC
      READ INPUT TAPE 5,110,(CIN(I),I=1,NSPEC)
      READ INPUT TAPE 5,110,(CINOX(I),I=1,NSPEC)
  110 FORMAT(7F10.0)
      WRITE OUTPUT TAPE 6,115,(CIN(I),I=1,NSPEC)
  115 FORMAT(34HOSPECIFIED C-FACTORS AT JUNCTION 1//
     *(1H 7F12.0))
      DO 1000 I=1,100
 1000 JPRT(I)=0
      READ INPUT TAPE 5,1001,NOPRT
 1001 FORMAT(I5)
      READ INPUT TAPE 5,1002,(JPRT(I),I=1,NOPRT)
 1002 FORMAT(14I5)
      WRITE OUTPUT TAPE 6,116,(CINOX(I),I=1,NSPEC)
  116 FORMAT(41HOSPECIFIED OXYGEN C-FACTORS AT JUNCTION 1//
     *(1H 7F12.2))
      WRITE OUTPUT TAPE 6,120,(N,CLEN(N),B(N),AREA(N),CN(N),QNET(N),
     *R(N),(NJUNC(N,K),K=1,2),N=1,NC)
  120 FORMAT(101H1CHANNEL     LENGTH      WIDTH       AREA      MANNING
```

FIGURE - 10

DYNAMIC WATER QUALITY PROGRAM

```
     *NET FLOW     HYD. RADIUS     JUNCTIONS AT ENDS//
     *(1H I5,F11.0,F8.0,F10.0,F9.3,F10.2,F13.1,I12,I6))

C**** INITIALIZATION

  130 DELTQ=DELT*FLOATF(NODYN)
      CDECAY=1.0-DECAY
      NTAG=0
      IPRT=NQPRT
      DO 358 N=1,NC
      IF(NJUNC(N,1)-NJUNC(N,2))358,358,357
  357 KEEP=NJUNC(N,1)
      NJUNC(N,1)=NJUNC(N,2)
      NJUNC(N,2)=KEEP
  358 CONTINUE
C********** JUNCTION VOLUMES AND SURFACE AREAS
      DO 373 J=1,NJ
      ASUR(J)=0.0
      VOL(J)=0.0
      DO 371 K=1,6
      IF(NCHAN(J,K))372,372,370
  370 N=NCHAN(J,K)
      ASUR(J)=ASUR(J)+B(N)*CLEN(N)
  371 VOL(J)=VOL(J)+(Q(N)/V(N))*CLEN(N)
  372 ASUR(J)=0.5*ASUR(J)
  373 VOL(J)=0.5*VOL(J)
      IF(SENSE SWITCH 1)374,377
  374 WRITE OUTPUT TAPE 6,375,(J,ASUR(J),VOL(J),J=1,NJ)
  375 FORMAT(35H1JUNCTION     SURFACE AREA     VOLUME//(1H I5,2E16.8))
      WRITE OUTPUT TAPE 6,376,(N,AREA(N),CLEN(N),B(N),N=1,NC)
  376 FORMAT(50H1CHANNEL     AREA        LENGTH       WIDTH       VMAX//
     *(1H I5,3E12.4))
  377 CONTINUE
C********** SALTS
      DO 378 J=1,NJ
      SALT(J)=C(J)*VOL(J)
  378 OXY(J)=COX(J)*VOL(J)
      IF(SENSE SWITCH 1)380,382
  380 WRITE OUTPUT TAPE 6,381,(J,SALT(J),OXY(J),J=1,NJ)
  381 FORMAT(20H1JUNCTION          SALT//(1H I5,2E20.8))
  382 CONTINUE
C**********EDDY DIFFUSION CONSTANT
      DO 385 N=1,NC
  385 DIFFK(N)=2.6208*R(N)*DELTQ/CLEN(N)
      IF(SENSE SWITCH 1)386,388
  386 WRITE OUTPUT TAPE 6,387,(N,DIFFK(N),N=1,NC)
  387 FORMAT(16H1CHANNEL    DIFFK//(1H I5,E20.8))
  388 CONTINUE

C*************************************************************************
C                        MAIN QUALITY LOOP
C*************************************************************************
      DO 480 ICYC=1,NQCYC

C*****READ SYSTEM CONDITIONS

  405 READ TAPE 3,ICYCTF,(YNEW(J),J=1,NJ),(Q(N),N=1,NC)
  406 IF(ICYCTF-NSTOP)409,407,407
  407 REWIND 3

C**********SALTS TRANSFERRED IN CHANNELS (INTO NEXT JUNCTION)

  409 DO 428 N=1,NC
      NL=NJUNC(N,1)
      NH=NJUNC(N,2)
  411 IF(Q(N))415,410,410
  410 J=NL
      GO TO 420
  415 J=NH
C********** SALT ADVECTION
  420 ADSALT=C(J)*Q(N)*DELTQ
      SALT(NH)=SALT(NH)+ADSALT
      SALT(NL)=SALT(NL)-ADSALT
C**********EDDY DIFFUSION (SALT)
      DISALT=DIFFK(N)*ABSF(Q(N))*(C(NL)-C(NH))
      SALT(NH)=SALT(NH)+DISALT
      SALT(NL)=SALT(NL)-DISALT
      IF(SENSE SWITCH 6)428,425
C********** OXYGEN ADVECTION
  425 ADOX=COX(J)*Q(N)*DELTQ
      OXY(NH)=OXY(NH)+ADOX
      OXY(NL)=OXY(NL)-ADOX
C********** OXYGEN DUFFUSION
      DIOX=DIFFK(N)*ABSF(Q(N))*(COX(NL)-COX(NH))
      OXY(NH)=OXY(NH)+DIOX
      OXY(NL)=OXY(NL)-DIOX
  428 CONTINUE

C**** DECAY AND MASS TRANSFER

      IF(SENSE SWITCH 6)500,534
C**********DECAY
  534 DO 535 J=2,NJ
      SALT(J)=SALT(J)*DECAY
      OXY(J)=OXY(J)-C(J)*VOL(J)*CDECAY
C**********MASS TRANSFER
  535 OXY(J)=OXY(J)+0.00000145*DELTQ*VOL(J)*(CS-COX(J))

C**********CORRECT JUNCTION VOLUME AND FIND NEW CONCENTRATION FACTOR

  500 NTAG=NTAG+1
      IF(NTAG-NSPEC)510,505,505
  505 NTAG=0
  510 C(1)=CIN(NTAG+1)
      COX(1)=CINOX(NTAG+1)
      COX(1)=CINOX(NTAG+1)
  513 DO 435 J=2,NJ
      IF(QIN(J))515,525,520
  515 SALT(J)=SALT(J)-CSPEC(J)*QIN(J)*DELTQ
      OXY(J)=OXY(J)-CSPEC(J)*QIN(J)*DELTQ
      GO TO 525
  520 SALT(J)=SALT(J)-C(J)*QIN(J)*DELTQ
```

539

FIGURE - 10, CONTINUED

```
      OXY(J)=OXY(J)-COX(J)*QIN(J)*DELTQ
  525 VOL(J)=VOL(J)+ASUR(J)*(YNEW(J)-Y(J))
      C(J)=SALT(J)/VOL(J)
      COX(J)=OXY(J)/VOL(J)
  435 CONTINUE

C**** PREVENT NEGATIVE CONCENTRATION AND SUPERSATURATION

      DO 441 J=1,NJ
      Y(J)=YNEW(J)
      IF(C(J))436,437,437
  436 C(J)=0.0
      SALT(J)=0.0
      WRITE OUTPUT TAPE 6,541,J,ICYC
  541 FORMAT(39HODEPLETION CORRECTION MADE AT JUNCTION I3,7H CYCLE I4)
  437 IF(COX(J))438,439,439
  438 COX(J)=0.0
      OXY(J)=0.0
  439 IF(COX(J)-CS)441,441,440
  440 COX(J)=CS
      OXY(J)=CS*VOL(J)
  441 CONTINUE
  445 CONTINUE

C**********************************************************************
C     PRINT QUALITY OUTPUT
C**********************************************************************
      IF(ICYC-IPRT)480,450,450
  450 IPRT=IPRT+NOPRT
      HOURS=DELTQ*FLOATF(ICYC)/3600.0
      KDAYS=HOURS/23.9999
      HOURS=HOURS-FLOATF(24*KDAYS)
      WRITE OUTPUT TAPE 6,451,ICYC,KDAYS,HOURS
  451 FORMAT(35H1SYSTEM STATUS AFTER QUALITY CYCLE I4,I12,6H DAYS,
     *F6.2,6H HOURS//
     *      107H JUNCTION      HEAD      CONC. FACTOR      OXYGEN CONC.
     * CHANNEL      FLOW      CONC. FACTOR      OXYGEN CONC.//
     *      103H  NUMBER       (FT)          (MGL)             (MGL)
     *  NUMBER     (CFS)          (MGL)             (MGL)///)
      DO 475 I=1,NOPRT
      J=JPRT(I)
      IF(SENSE SWITCH 6)452,453
  452 WRITE OUTPUT TAPE 6,454,J,Y(J),C(J)
      GO TO 455
  453 WRITE OUTPUT TAPE 6,454,J,Y(J),C(J),COX(J)
  454 FORMAT(1H0I5,F12.2,F14.1,F16.2)
  455 DO 473 K=1,6
      IF(NCHAN(J,K))475,475,456
  456 N=NCHAN(J,K)
      IF(J-NJUNC(N,1))460,457,460
  457 FLOW=Q(N)
      IF(Q(N))459,458,458
  458 CZ=C(J)
      CO=COX(J)
      GO TO 465
  459 JOPP=NJUNC(N,2)
      CZ=C(JOPP)
      CO=COX(JOPP)
      GO TO 465
  460 FLOW=-Q(N)
      IF(Q(N))461,461,462
  461 CZ=C(J)
      CO=COX(J)
      GO TO 465
  462 JOPP=NJUNC(N,1)
      CZ=C(JOPP)
      CO=COX(JOPP)
  465 CONTINUE
      IF(SENSE SWITCH 6)466,467
  466 WRITE OUTPUT TAPE 6,468,N,FLOW,CZ
      GO TO 473
  467 WRITE OUTPUT TAPE 6,468,N,FLOW,CZ,CO
  468 FORMAT(1H I6I,F12.0,F13.1,F16.2)
  473 CONTINUE
  475 CONTINUE
  480 CONTINUE

C*****PREPARE FOR NEXT CASE AND RECYCLE

      REWIND 10
      REWIND 3
      PUNCH 103,(ALPHA(I),I=1,48)
      PUNCH 471,(J,C(J),CSPEC(J),COX(J),OSPEC(J),J=1,NJ)
  471 FORMAT(I5,4F10.3)
      WRITE OUTPUT TAPE 6,470
      PRINT 470
  470 FORMAT(24HOEND OF QUALITY PROGRAM.)
      GO TO 100
      END(1,1,0,0,0,0,1,0,0,1,0,0,0,0,0)
```

530

FIGURE - 10, CONTINUED

PROGRAM APPLICATION

Preliminary Studies

As the computer program was developed and each of the various processes was incorporated, the program was subjected to continuous testing. This testing or "debugging" procedure was carried out on several simple geometries in order to avoid the complexity of the Delta system which was not only costly to simulate but might have obscured some peculiarities of the proposed models.

Two types of simple systems were studied, an estuary comprised of 29 channel segments connected end-to-end and a more complex "pseudo-delta" comprised of 27 segments connected at 24 junctions in a triangular pattern. These systems were subjected to tidal action at one boundary point and received flows with or without pollutant load at selected locations. As in the case of the Delta system itself, each of these was solved for the dynamic steady state hydraulic behavior before attacking the pollution problem.

A variety of special problems were treated in these systems. They are summarized as follows:

Linear Estuary:

1. Intrusion of salinity from a constant source by advection only.
2. Intrusion of salinity from a variable source by advection only.
3. Intrusion of salinity by diffusion only (constant source)
4. Intrusion of salinity by advection and diffusion combined (variable source).
5. Advection of an upstream pollutant through the estuary.
6. Degradation of an upstream pollutant advected and diffused through the estuary.
7. Reoxygenation of an oxygen deficient estuary with an organic load advected and diffused from an upstream source.

Pseudo-Delta:

1. Intrusion of salinity from a constant source by advection only.
2. Intrusion of salinity from a variable source by advection only.

3. Intrusion of salinity from a variable source by advection and diffusion combined.

These studies were designed to prove each element of the water quality model separately and then in conjunction with the other processes. In addition, during the course of the investigation particular attention was given to the effect of boundary conditions, such as the salinity at the lower extremity of the Delta, and to peculiarities of the numerical methods such as the time of convergence, stability, and computational mixing effects. Results of these studies led to the modification and refinement of the model preparatory to testing on the entire Delta system.

In order to illustrate the testing procedure and some of the special problems encountered a brief discussion is included herein of some selected cases studied with the "linear estuary" and one test run with the "pseudo-Delta".

Linear Estuary - Test Cases

The linear estuary was formed of 29 segments each with identical properties as follows: length = 4000 feet, width = 500 feet, depth = 20 feet, area normal to flow = 10,000 square feet and Manning's n = 0.030. Channel segments were connected end-to-end forming 30 junctions which were assigned volumes equal to the half-volumes of the connecting channel segments.

A predetermined wave form with a range in amplitude from +1.50 to -1.56 and a period of 12.5 hours was imposed on the lower boundary of the estuary while flows ranging up to 5000 cfs were introduced at the upper end. As an example of the estuary's response; when 5000 cfs was admitted at the upper end current reversals occurred as far upstream as 20 channel segments (80,000 feet). At the lower end of the estuary velocities ranged from 1.01 fps upstream to 1.56 fps downstream. At the upper most channel segment the range was from 0.45 to 0.56 fps, always downstream. When the inflow was reduced to 500 cfs reversals were experienced in all but the most upstream segment. Velocities at the lower end ranged from 1.23 seaward to 1.30 landward while at the upper end they ranged from 0.02 to 0.07 seaward.

Two examples of test cases have been selected for inclusion in the report in order to illustrate the behavior of this relatively simple system and the various

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processes included in the water quality model.

Figure 11 depicts the advance of salinity into the estuary due to the mechanisms of advection and diffusion. The significant features of this test case are the following:

1. Salt intruded into the estuary a distance of more than 80,000 feet. After 500 hours of operation with 3000 ppm at the mouth of the estuary the salt concentration at the 80,000-foot point reached 109 ppm.
2. Salinity fluctuated widely at various stations along the estuary, most notably near the mouth. At station 2, 4000 feet from the mouth concentrations ranged from about 500 to more than 2900 ppm as illustrated in the inset in Figure 11.
3. Salinity fluctuations at station 2 were approximately 2.5 hours out-of-phase with velocity fluctuations but were nearly in-phase with the water level fluctuations.
4. After 200 hours of operation there was little significant change in the pattern of salinity in the lower part of the estuary although salt continued to move upstream at the upstream stations. Between 200 and 500 hours (24 tidal cycles) the mean salt concentration at the mid-point of the estuary almost doubled. This was attributed largely to the diffusion mechanism which continued to operate and had not established complete equilibrium with advective transport as stated in equation 11.

Figure 12 shows the behavior of the estuary under dynamic conditions while receiving a continuous discharge of sewage (20 ppm 5-day, 20°C BOD). It was assumed for this case that the estuary was initially free of organic loads and was completely saturated with oxygen (9.17 ppm at 20°C.) The significant features of this test case are the following:

1. Equilibrium between deoxygenation and reaeration was rather quickly established. The low point on the sag curve changed only slightly after about 500 hours.
2. An initial deficit of about 1.7 ppm occurred at the junction receiving the discharge.
3. The recovery limb of the sag curve continued to change under the combined influence of advective and diffusive transport and the mechanisms of decay and reaeration.

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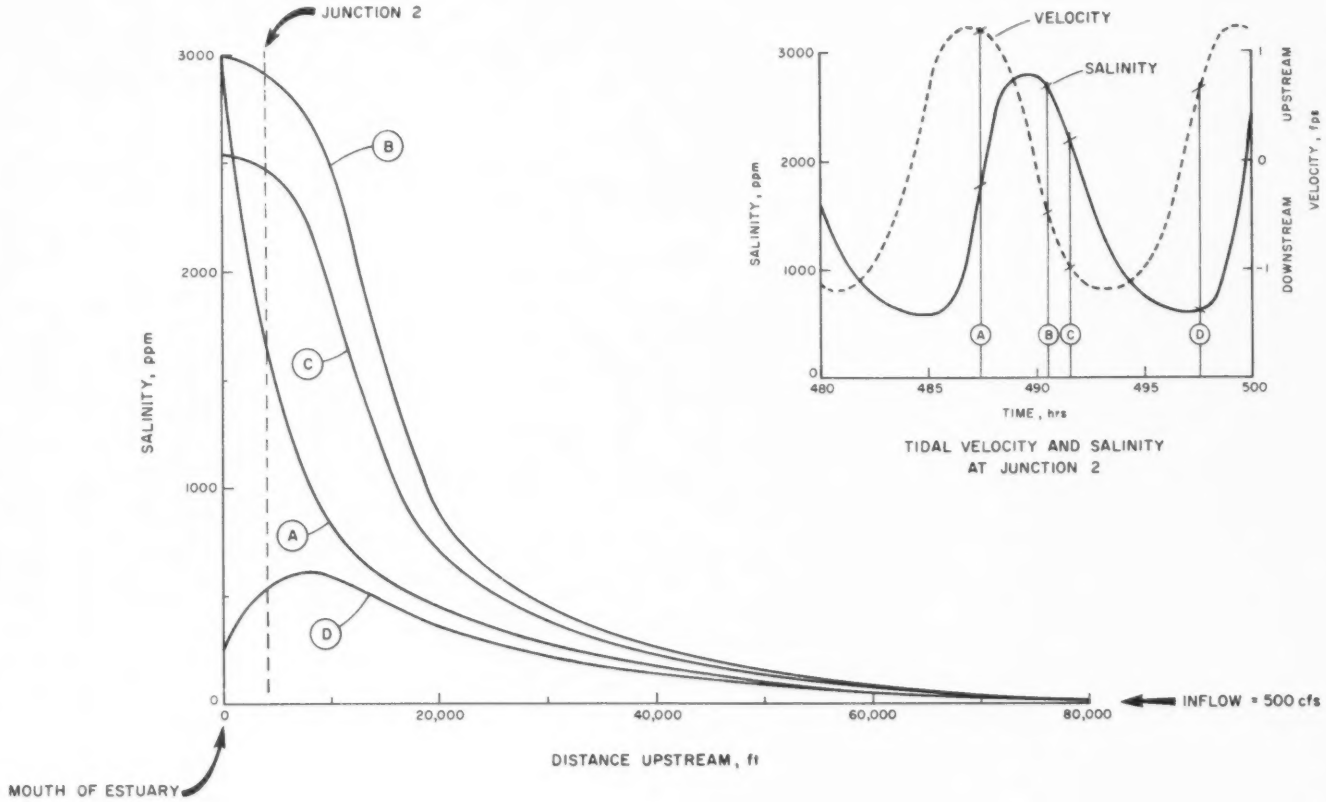

FIGURE 11

SALINITY INCURSION INTO A LINEAR ESTUARY

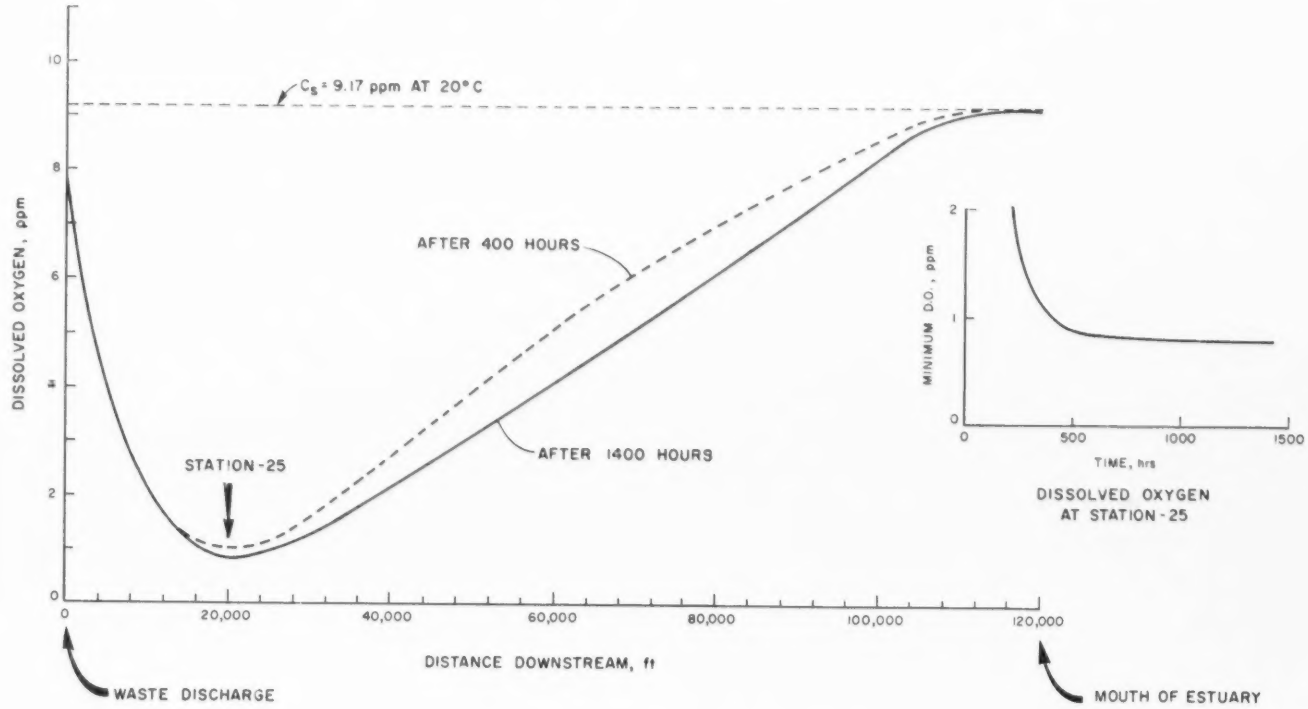

FIGURE 12

OXYGEN SAG – LINEAR ESTUARY

4. The constant boundary oxygen concentration at the mouth of the estuary tended to support slightly higher oxygen concentrations in the lower estuary.

Pseudo-Delta - Test Case

Figure 13 shows the results of a special test case run on the so-called "Pseudo-Delta", a hypothetical system of channels organized in the pattern of equilateral triangles. The system consists of 27 channel segments, 3 for each major channel, connected at 24 junctions. Two apexes of the principal triangle serve as flow input locations,somewhat analogous to the two major rivers of the Sacramento-San Joaquin Delta, and the other serves as the tide input location. Channel lengths, widths, depths, cross-sections, and roughnesses were varied in a somewhat arbitrary way to obtain an asymmetric configuration comparable to the real Delta. In the same manner as for the linear estuary a hydraulic solution was obtained first for a given wave form imposed on the system and then the water quality problem was initiated.

The example given in Figure 13 supposes that flows of 1000 cfs are introduced at the upper corners with salinity concentration of 500 and 750 ppm. A salinity of 3000 ppm was maintained at the lower corner.

The behavior of the system for the last of eight tidal cycles to which it was subjected is illustrated in the figure. At t = 88 hours (A) the tide is flooding, reaching its peak at about 93.5 hours as may be seen in B and C. Subsequently, ebbing flows flush the estuary except for its lower extremity. Gradients are established between the several contributing sources pending the next cycle.

This simple system proved to be an instructive step toward development and verification of the model for its final application to the Delta System.

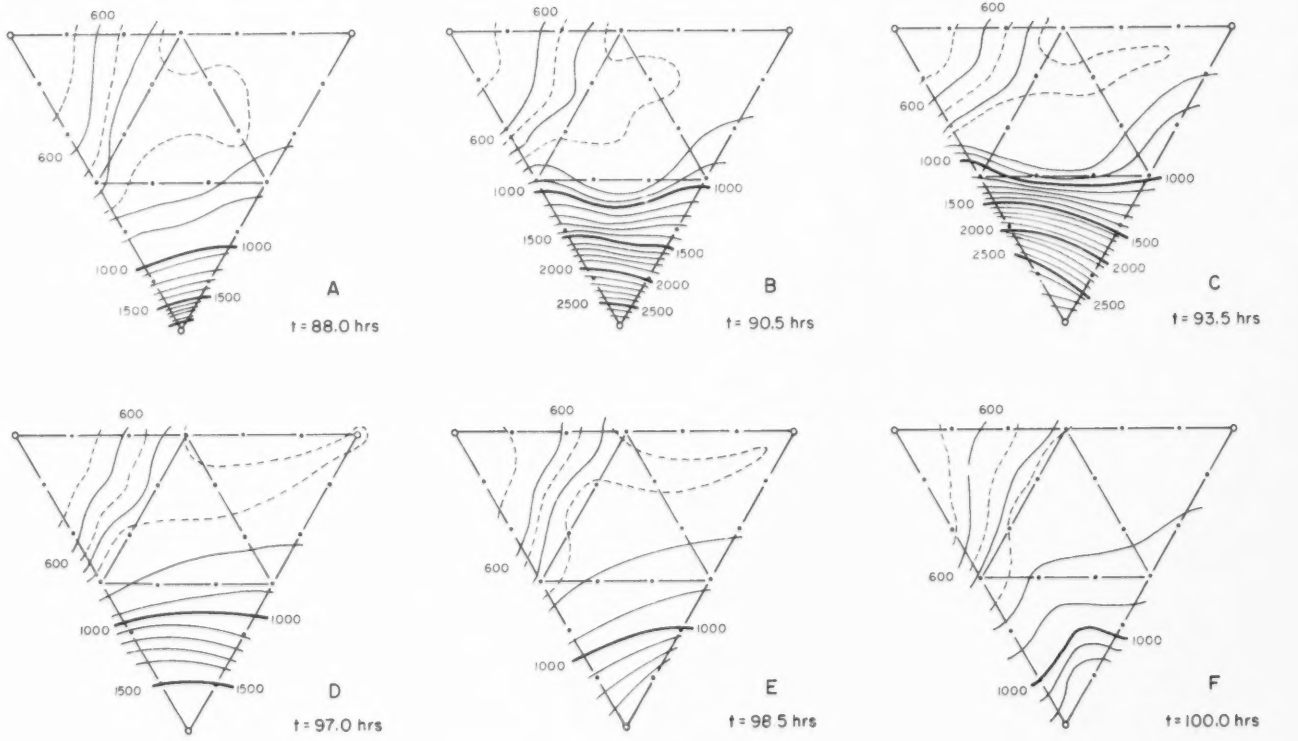

FIGURE 13

SALINITY INCURSION INTO THE
PSEUDO – DELTA SYSTEM

CASE STUDIES - SACRAMENTO-SAN JOAQUIN DELTA

Conservative Pollutant

While there are a number of interesting conservative pollutant problems which the model might be capable of solving, it was thought that a simple influence pattern produced by the proposed San Joaquin Master Drain would be both informative and timely. The hydraulic conditions which include the influence of this discharge had already been evaluated (See Chapter III, Figure 6) and all the basic data was on tape ready for use in the quality program.

Boundary conditions specified for the study were quite simple:

Master Drain discharge	=	900 cfs
Concentration, drain inflow	=	3000 ppm TDS
Concentration, all other flows	=	0 ppm
Initial conc., all locations	=	0 ppm

Both advective and diffusional processes were operative in the model. Exports occurred at concentrations produced at the junctions during the course of the run. The only TDS import was, of course, that of the drain located near Antioch on the San Joaquin River.

Figures 14 and 15 summarize the condition of the Delta System after about 30 days of prototype operation. They represent, respectively, equilibrium conditions for maximum and minimum changes in water quality throughout the Delta channels. The differences between the two figures reflect the effect of tidal oscillation under dynamic steady state conditions. For purposes of clarity in presentation iso-TDS lines are drawn over the Delta network.

It is not within the scope of this report to interpret the results obtained except as they exemplify the behavior of the water quality model. A particularly important point to note in this connection is that the figures shown are in the nature of influence diagrams, that is, they tend to show the area affected and magnitude of the effect of only <u>one</u> component of inflow. The contributions of other inflows can be separately evaluated and by superposition the combined effects can be determined. All results must be for <u>identical hydraulic conditions</u>.

FIGURE 14

SAN JOAQUIN MASTER DRAIN INFLUENCE - MAXIMUM TOTAL DISSOLVED SOLIDS (TDS) CONCENTRATIONS

(AUGUST - 1964)

58a

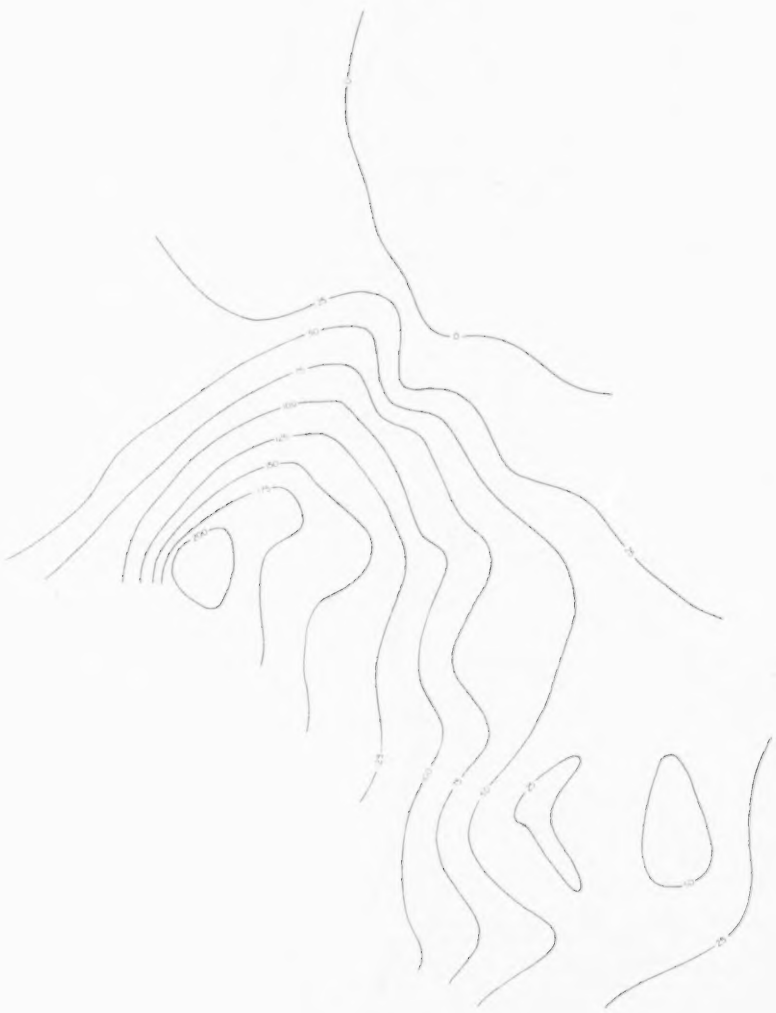

FIGURE 15

SAN JOAQUIN MASTER DRAIN INFLUENCE - MINIMUM
TOTAL DISSOLVED SOLIDS (TDS) CONCENTRATIONS
(AUGUST - 1964)

586

There are no obvious anomalies in the results obtained with the water quality program in this test case. Behavior is essentially consistant with that anticipated from the hydraulics of the system. Intrusion of solids occurred in those channels where current reversals were experienced; it did not occur, for example, in the northern Delta where flows were always seaward. Some pollution was drawn southward because of the large diversions near Tracy. The southeast Delta, with its low inflow, experienced significant changes in quality due to the Antioch discharge. All these occurrances are consistant in themselves and are reasonable to anticipate under the conditions stipulated. Although there are no direct means of verifying the model's ability to exactly predict a future condition, it seems appropriate to conclude that it constitutes a superior engineering tool with which to study, evaluate, and compare alternatives for management of Delta water quality.

Non-Conservative Pollutant

Evaluating the impact of domestic sewage discharge on the oxygen resources of the Delta network under dynamic steady state conditions approaches the ultimate test of the model's veracity. Such a test requires that all processes, advection, diffusion, decay, reaeration, and import-export be simultaneously operative.

As a case study, both for verification purposes and because of its significance in Delta water quality control, the discharge of sewage from the City of Stockton was made the subject of a brief study. Assuming a steady discharge into the San Joaquin River at Stockton of 40 cfs of sewage the model was operated for a period of 10 days. At the end of this time the Delta oxygen resources, assumed to have been initially at 8.38 ppm (saturation at 25^{o} C), were at near equilibrium with the waste load. Figure 16 illustrates the response of the Stockton ship canal and its immediate tributaries to several hypothetical waste loadings. The several curves were determined from a single study.

While it is acknowledged that there are many assumptions which must be made in order to set up this problem (reaction velocities, diffusion coefficients, temperature effects, etc.) the results obtained in this preliminary study support the conceptual correctness of the model. A more or less typical "sag" occurs and the expected fluctuations in oxygen concentration are noted. Areas of the Delta remote

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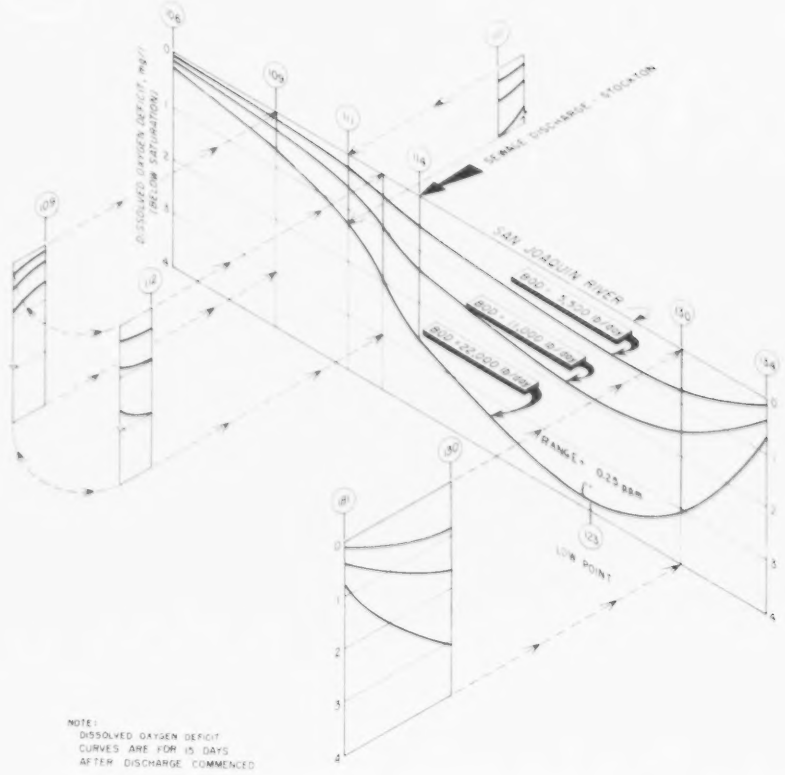

LOCATION MAP
(SACRAMENTO SAN JOAQUIN RIVER DELTA)

59a

FIGURE 16

OXYGEN SAG CURVES
SAN JOAQUIN RIVER NEAR STOCKTON
(AUGUST - 1964)

from Stockton are not influenced by the organic load and recovery was experienced in the classical manner, even though somewhat complicated by the unusual geometry and dynamics of the system.

It will be important in the future, as additional information on photosynthetic oxygenation is obtained, to include this effect in the model. The basic structure is developed and the essential mechanisms are now included. Additional sophistication should be achieved slowly, consistant with need and understanding of the complex behavior of the Delta. It is believed that the present model can serve well to implement investigations, while it too is being continually improved.

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V SUMMARY AND CONCLUSIONS

The investigation described in this report was aimed at the development of a practical tool for the study of water quality problems of the Sacramento - San Joaquin Delta, a complex system of estuarial channels, which is a key element in the California Water Plan. Through the Delta region water will be transferred from Northern California supplies to the California Aqueduct for delivery to water deficient areas of the south. It is expected that this transfer and the discharge of agricultural and domestic waste waters will greatly influence water quality and beneficial water use in the area.

The U. S. Public Health Service, because of its responsibilities in the field of water supply and pollution control, sponsored the development of a model of the Delta system which could serve its needs in studying water quality problems. It was required that the model have the capability of reproducing the hydraulic behavior of the Delta under tidal action and that it correctly depict the temporal and spacial disposition of conservative and non-conservative pollution.

Hydraulically, the Delta System is so complex as to defy traditional mathematical analysis. It demands representation in some model form, physical, analog, or mathematical. For reasons of cost, time of development, and complexity it was considered that a mathematical model set up for finite difference solution with the digital computer would be the most practical.

A model of this type, designed by Water Resources Engineers, Inc. of Lafayette, California, was formed of two compatible components, one which determines the hydraulic behavior of the system and another which determines the fate of conservative or non-conservative pollutants circulated within the system. The principal features of each of these components and some of the special problems related to their development are described in detail in this report and are summarized briefly in this concluding chapter.

Hydraulic representation of the prototype was accomplished by reducing it to a system of one-dimensional channels which were assigned pertinent hydraulic properties related to flow. Volume, surface area, and depth characteristics were

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assigned to channel junctions. As it was finally reduced the model consisted of 625 channel segments connected at 572 junctions. Tidal elevations, velocities, and discharges for all elements of the system were determined for specified hydrologic and tidal conditions by solving the one dimensional equations of motion and continuity.

The validity of the model was confirmed by comparison of predicted tidal elevations with observed elevations on the prototype. Both phase and amplitude of fluctuations were closely reproduced . Current motions were confirmed qualitatively, although field data was meager.

The water quality component of the Delta Model was designed to represent the transport processes of advection, advective dispersion, and eddy diffusion. Quality constituents were transferred by these mechanisms between system junctions in accordance with properties assigned to channel segments.

Test cases, to verify the functional validity of the model, were performed on simplified estuarial systems. A linear channel of 29 segments and a Delta-configuration of 27 segments were subjected to hypothetical hydrologic and tidal conditions and hydraulic solutions were obtained. Results of these solutions were supplied as input for investigation of water quality changes. Each of the processes affecting movement of a conservative pollutant within these systems were examined; first, separately and then in conjunction with each other. In this manner the model was progressively checked, modified, and verified as to its capability to reproduce faithfully the processes acting in the prototype.

In order to illustrate further the model's applicability for study of conservative pollution, a preliminary investigation of the influence of the San Joaquin Master Drain on Delta water quality was carried out. The study, performed under assumed future project conditions, illustrated the effect of Southern Delta withdrawals in the circulation of pollution and the nature of dynamic effects due to tidal action.

In addition to the transfer mechanisms essential to determining the fate of conservative pollutants in the Delta System, the model was adapted to the study of non-conservative pollution. Decay of a pollutant was represented in the form of the monomolecular reaction, traditionally applied to biodegradation of organic material. Reaction velocities were taken as constant throughout the system but

provision was made for both temporal and spacial variation.

In order to treat the classic "oxygen sag" problem the reaeration mechanism was represented in the model. A variety of methods for estimating the reaeration coefficient as a function of the system's hydraulic properties were explored. In initial studies the reaeration coefficient was treated as constant in time and space.

A brief study was also performed to verify, on the larger scale, the model's usefullness for investigation of organic pollution. The oxygen sags resulting from several assumed loadings of domestic sewage introduced near the City of Stockton were produced by the model. The action of tides and withdrawals of water in the Southern Delta influenced the shape of the sag curves and the location and persistance of zones of oxygen deficiency.

It is concluded that the model which finally resulted from this investigation will serve as a valuable tool for the study of water quality problems in complex estuarial systems. In its present form the model, consisting of two components, hydraulic and water quality, is capable of application to the following general types of water quality problems:

1. Intrusion of salinity.
2. Routing of quality constituents from upstream sources.
3. Concentration of salts by evaporation, consumptive use, and irrigation reuse and drainage.
4. Flushing of pollution by upstream releases.
5. Diffusion of pollutants due to tidal action.
6. Dispersion and degradation of non-conservative pollutants.

The Delta Water Quality Program written for the IBM 7094 computer is structured in such a way that it may be easily modified to provide for other processes which bear on water quality. As the need arises to consider more complicated problems, such as those related to temperature change, photosynthesis, and sediment transport, the program can be supplemented. The potential for practical use of this engineering tool is limited only by the ability of the designers to correctly describe the problems to be solved and by the availability of adequate field information to

support the model's veracity. Considerable effort needs to be expended in acquiring a better understanding of the real environment of problems and the practical ways for attaining useful solutions. The present model should contribute significantly toward this end.